

## Report stiintific

Decembrie 2015– Septembrie 2016

### Thermodynamic Properties and Correlation Functions of Low-Dimensional Ultracold Gases

#### Team

Ovidiu I. Patu CS III (Investigator principal), Cecil Pompiliu Grunfeld CS I

#### Results

##### ISI Articles

[PK3] Ovidiu I. Patu and Andreas Klumper, *Thermodynamics, contact and density profiles of the repulsive Gaudin-Yang model*, Phys. Rev. A, **93**, 033616 (2016) [Impact factor (2015) 2.765 , Top 25% Optics]

##### Articles in preparation

[PK4] Ovidiu I. Patu and Andreas Klumper, *Thermodynamics and local correlation functions of the Bose-Fermi mixture*

##### Conference talks

[C3] Ovidiu I. Patu and Andreas Klumper, [Efficient thermodynamic description of the Gaudin-Yang model](#), talk presented at [Recent Advances in Quantum Integrable Models \(RAQIS 2016\)](#) , August 22-26, 2016 Geneve (Switzerland)

[C4] Ovidiu I. Patu and Andreas Klumper, [Efficient thermodynamic description of the Gaudin-Yang model](#), talk presented at [New Trends in Low-Dimensional Physics: Quantum Integrability and Applications](#) September 1-15, 2016 Beijing (China)

#### Scientific activities

**Thermodynamics and local correlation functions of the Bose-Fermi mixture** The one-dimensional mixture of bosons and fermions interacting via a delta-function potential is also Bethe ansatz solvable. It can be shown that the solutions of the Bethe ansatz equations are all real which means that in principle the application of the TBA method should produce a finite number of equations. Using the same method employed for the 2CFG and 2CBG we have obtained a system of two NLIEs characterizing the Bose-Fermi mixture which has the correct behavior in the Tonks-Girardeau regime and also reproduces the well-known results in the noninteracting limit. The result obtained has a similar structure with the one obtained in the case of the two-component Bose and Fermi gas. The grandcanonical potential is

$$\phi(\mu_B, \mu_F, T) = \frac{-T}{2\pi} \int \ln(1 + a_1(k)) + \ln(1 + a_2(k)) dk$$

where  $\mu_F, \mu_B$  are the chemical potentials for bosons and fermions and the auxiliary functions  $a_{1,2}$  satisfy the following system of nonlinear integral equations

$$\ln a_1(k) = -\frac{(k^2 - \mu_B)}{T} + \int_{\mathbb{R}} K_0(k-k') \ln(1+a_1(k')) dk' + \int_{\mathbb{R}-i\epsilon} K_2(k-k') \ln(1+a_2(k')) dk'$$

$$\ln a_2(k) = -\frac{(k^2 - \mu_F)}{T} + \int_{\mathbb{R}+i\epsilon} K_1(k-k') \ln(1+a_1(k')) dk'$$

$$K_0(k) = \frac{1}{2\pi} \frac{2c}{k^2 + c^2} \quad K_1(k) = \frac{1}{2\pi} \frac{c}{k(k+ic)} \quad K_2(k) = \frac{1}{2\pi} \frac{c}{k(k-ic)}$$

This system of equations reproduce the well-known results in the literature in the non-interacting limit and Takahashi's result in the Tonks-Girardeau regime. In this stage we have also derived a set of thermodynamic identities for multi-component systems with contact interactions known as Tan's relations. In the course of the derivation we have used the fact that for delta-function potentials the wave function presents a discontinuity in the derivative which produces a  $1/k^4$  tail of the momentum distribution. We have also checked that our results for the Bose-Fermi mixture satisfy this set of identities and obtained perfect agreement.

The system of equations presented above was used for the computation of the thermodynamic properties of the system (densities, specific heat, compressibility, susceptibility, etc.) and the contact which governs the large momentum distribution of the system. As a function of the coupling strength the contact is a nonmonotonous function (presents a local maximum) at low values of the reduced temperature a situation which echoes a similar phenomenon present in the 2CFG. In addition the contact presents a pronounced minimum (which also depends on the boson fraction) at strong coupling and low-temperatures which signals a significant reconstruction of the momentum distribution. This shows that the Bose-Fermi mixture presents two different temperature scales with different quantum behaviours. In one interval the Tomonaga-Luttinger liquid theory is valid but as we increase the temperature we reach a regime which can be imagined as the equivalent of the spin-incoherent regime present in the 2CFG. The transition between this two regimes manifests as the local minimum of the contact. These results which will be published in [PK4] (most probably after the termination of the contract) will of course acknowledge the funding source.

Some of the results obtained during the contract were also presented at two top conferences in the field of integrable systems [C3], [C4]. As a result several collaborations with international researchers were initiated.

Principal investigator,

Ovidiu I. Patu