Scientific report

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Thermodynamic Properties and Correlation Functions of Low-Dimensional Ultracold Gases

Team

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Results

ISI Articles

[PK1] Ovidiu I. Patu and Andreas Klumper, *Correlation lengths of the repulsive one-dimensional Bose gas*, Phys. Rev. A, **88** 033623 (2013), [Impact factor (2012) 3.046 , Top 25% Optics]

[KP1] Andreas Klumper and Ovidiu Patu, *Temperature-driven crossover in the Lieb-Liniger model*, Phys. Rev. A, **90** 053626 (2014), [Impact factor (2013) 2.991 , Top 25% Optics]

[P1] Ovidiu I. Patu, *Correlation functions and momentum distribution of one-dimensional hard-core anyons in optical lattices*, accepted for publication in J. Stat. Mech. [Impact factor (2013) 2.056, Top 25% Mathematical Physics] [arXiv:1409.2321]

Articles in preparation

[PK2] Ovidiu I. Patu and Andreas Klumper, *Thermodynamics, density profiles and correlation functions of the trapped one-dimensional two-component Bose gas*

[PK3] Ovidiu I. Patu and Andreas Klumper, *Efficient thermodynamic description of the 1D two-component Fermi gas. I. The repulsive case*

[PK4] Ovidiu I. Patu and Andreas Klumper, *Efficient thermodynamic description of the 1D two-component Fermi gas. II. The attractive case*

Conference talks

[C1] Ovidiu I. Patu and Andreas Klumper, *Correlation lengths of the Lieb-Linger model*, talk presented at „Correlation Functions of Quantum Integrable Models”, 4-6 Sep 2013 Dijon (France)

[C2] Ovidiu I. Patu and Andreas Klumper, *Temperature driven crossover in the 1D Bose gas*, talk presented at „Integrable Lattice Models and Quantum Field Theories”, 28 June- 2 July, 2014 Bad Honnef (Germany)

Participation at conferences

[C3] Ovidiu I. Patu Recent Advances in Quantum Integrable Systems, 1-5 September, 2014 Dijon (France)

Scientific activities

Correlation lengths and temperature-driven crossover in the Lieb-Liniger model. Introduced in the 1960's, the Lieb-Liniger model, which describes one-dimensional bosons interacting via a delta-function potential, is arguably one of the most investigated solvable models. Even though the initial theoretical investigations were considered as academic exercises, the recent progress in the trapping and cooling of atomic gases made possible the experimental realization of such systems. These experimental advances highlighted the need for clear theoretical predictions to be compared with experimental data.

The correlation functions play an important role in the characterization of any physical system being experimentally accessible through a plethora of methods: interference, analysis of particle losses, photoassociation, Bragg and photoemission spectroscopy, density fluctuation statistics, etc. Even though the Lieb-Liniger model is integrable the complete characterization of the correlation functions is still an open problem. The most complete results were obtained in the Tonks-Girardeau limit (infinite repulsion between the particles). In this limit the asymptotic behaviour of the correlators can be rigorously derived using the solution of an associated Riemann-Hilbert problem results that confirmed the Conformal Field Theory/ Tomonaga-Luttinger Liquid (CFT/TLL) predictions at low-temperature.

In the case of finite repulsion the results in the literature are very scarce. In this case, together with A. Klumper we have derived the asymptotic behavior of the static correlation functions using a new method based on the Quantum Transfer Matrix (QTM) which makes use of the connection between the Lieb-Liniger model and the XXZ spin chain. An extensive numerical analysis of the correlation lengths was presented in [PK1] (partially funded by the project) and [C1]. This analysis revealed in the case of the density correlator an extremely interesting crossover phenomenon. Our findings can be summarized as follows. For all strengths of interactions the leading correlation length develops a nonzero imaginary part for temperatures larger than a critical temperature. As we approach the Tonks-Girardeau limit we find two additional crossovers in which the leading correlation lengths successively change places as the dominant one. Interestingly, the crossover phenomena happen when leaving the low-temperature dense phase either by increasing the temperature or by decreasing the density. These results were published in [KP1] and presented at [C2].

Correlation functions of 1D anyons in optical lattices. The experimental advances mentioned above have also opened the way for the possible experimental validation of 1D anyons (particles whose statistics interpolates continuously between fermions and bosons). In [P1] we have initiated a systematic study of the correlation functions of a system of hard-core 1D anyons which can be understood as the extension to arbitrary statistics of hard-core bosons on a lattice. Using the summation of form factors we have derived Fredholm determinant representations for the temperature-dependent static and dynamic correlation functions. Contrary to a long held belief these determinants can be evaluated numerically in an extremely efficient fashion using the Nystrom method with the Gauss-Legendre quadrature which allowed us to obtain extremely precise numerical data (errors of 10^{-10}). In the case of static correlation functions we derived analytical formulas for the large-distance asymptotic behavior and computed the momentum distribution which is experimentally accessible.

Density profiles and local correlation functions in the two-component Bose gas. The thermodynamic behavior of the two-component Bose gas with delta interaction (2CBG) can be obtained using the Thermodynamic Bethe Ansatz (TBA). However, this method produces an infinite system of nonlinear integral equations (NLIEs) which makes the extraction of relevant physical information extremely difficult. An efficient thermodynamic description of this model (only two NLIEs) was derived by A. Klumper and PI in [KP0]. In a typical experiment the gas is cooled in a magneto-optical trap whose presence can be effectively simulated by the presence of a harmonic potential in the Hamiltonian of the system. Even though the models considered in this project are integrable only in the homogeneous case (no external potential) the thermodynamic behavior (and some local correlation functions) of the systems in the presence of the harmonic potential can be obtained using the results from the homogeneous case coupled with the local density approximation (Thomas-Fermi). For the 2CBG we have used this approximation and the NLIEs obtained in [KP0] to numerically compute the density profiles of the trapped gas. Our results show that for a large region of the relevant parameters (chemical potential, magnetic field and temperature) there is no phase separation but it cannot be completely excluded. The region of uncertainty is H~0 and T~0 where due to the ferromagnetic nature of the groundstate the NLIEs become numerically unstable. Discussions with the participants at the RAQIS 2014 [C3] conference revealed a gap in the literature concerning the second order correlation function g_2. This local correlation function is extremely important being experimentally accessible but it can also be calculated from the grandcanonical potential of the system using the Helmann-Feynman theorem. We implemented a new numerical code for the calculation of this correlation function in both cases (homogeneous and inhomogeneous) obtaining results which together with the density profiles will be published in [PK2].

Thermodynamics of the two-component Fermi gas. Applying the QTM method for the two-component Fermi gas with delta interaction (2CFG) requires the identification of a lattice model which in a specific scaling limit produces the continuum model. For the 2CFG we have identified the lattice model as the $U_q(sl(2|1))$ (super) spin chain with a specific grading. The repulsive case is obtained performing the continuum limit from the critical region (|cos q|<1) while the attractive case is obtained performing the limit from the massive region (cosh q>1). The grandcanonical potential of the system is derived from the largest eigenvalue of the QTM of the spin chain (we should mention that the identification of the largest eigenvalue is a nontrivial task requiring the full exploration of the Bethe roots and accompanying holes). The result obtained for the grandcanonical potential is extremely simple :

$$\phi(\mu, H, T)=\frac{-T}{2\pi}\int \ln(1+a_1(k))+\ln(1+a_2(k))\,dk$$

where μ, H, T the chemical potential, magnetic field and temperature and the auxiliary functions $a_{1,2}$ satisfying the system of equations

$$\ln a_1(k)=-\frac{(k^2-\mu-H)}{T}\pm\int_{\mathbb{R}-i\epsilon} K_2(k-k')\ln(1+a_2(k'))\,dk' \quad K_2(k)=\frac{1}{k(k-ic)}$$

$$\ln a_2(k)=-\frac{(k^2-\mu+H)}{T}\pm\int_{\mathbb{R}+i\epsilon} K_1(k-k')\ln(1+a_1(k'))\,dk' \quad K_1(k)=\frac{1}{k(k+ic)}$$

The plus (minus) sign corresponds to the attractive (repulsive) case. The efficiency of the QTM should become obvious if we take into account that in this case TBA produces an infinite number of NLIEs which are extremely hard to implement numerically. Similar to the bosonic case in addition to the thermodynamic properties of the homogeneous system (pressure, magnetization, susceptibility) we have also computed the density profiles (for the trapped system). These results will be published in [PK3] (repulsive case) and [PK4] (attractive case).

At the end of Stage II (halfway through the duration of the project) the results funded are: 3 accepted or published ISI articles in top tier journals (top 25%) [PK1,KP1,P1], two presentations at international conferences [C1,C2] and three articles in advanced stages of preparation (75-90%) [PK2,PK3,PK4]. Taking into account that the estimated results for the entire duration of the project were 4 articles in ISI journals (top 50%) and 5 presentations at national or international conferences we consider that all the intermediary objectives were fulfilled.

[KP0] Andreas Klumper and Ovidiu I. Patu, Phys. Rev. A, **84** 051604(R) (2011) .

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