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Correlation functions and momentum distribution of one-dimensional hard-core anyons in optical lattices

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Abstract. We address the problem of calculating the correlation functions in a system of one-dimensional hard-core anyons that can be experimentally realized in optical lattices. Using the summation of form factors we have obtained Fredholm determinant representations for the time-, space-, and temperature-dependent Green's functions which are particularly suited to numerical investigations. In the static case we have also derived the large distance asymptotic behavior of the correlators and computed the momentum distribution function at zero and finite temperature. We present extensive numerical results highlighting the characteristic features of one-dimensional systems with fractional statistics.

Keywords: correlation functions, quantum integrability (Bethe Ansatz), solvable lattice models

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Contents

1. Introduction

One-dimensional (1D) systems possess certain characteristics which make them extremely interesting to theoreticians and experimentalists alike. From the theoretical point of view they are special because their simplicity makes them amenable to exact solutions which, contrary to a naive expectation, are characterized by extremely rich physics. Furthermore, in recent years numerous experimental realizations of quasi-one-dimensional materials opened the way for the investigation of properties not present in the 3D world. One such remarkable feature found only in low-dimensional systems is the presence of anyons [1–3], particles with statistics interpolating between fermions and bosons. One-dimensional anyons [4–38] represent an active area of research with an increasing literature on the subject partly motivated by several proposals [39–43] for experiments designed to find signatures of fractional statistics in ultracold gases.

The model studied in this paper [19–22] can be viewed as the generalization to arbitrary statistics of hard-core bosons on the lattice. Unlike the continuum analog, which is represented by impenetrable Lieb–Liniger anyons [4–18] this model has not been subjected to a thorough analytical investigation of the correlation functions, especially at finite temperature. The need for such calculations has become even more pressing in the last decade as a result of the plethora of experimental techniques developed for the measurement of correlators. Here, we fill this gap in the literature, deriving Fredholm determinant representations for the time-, space-, and temperature-dependent Green's functions. These representations not only represent the starting point for the rigorous investigation of the asymptotic behavior but also can be used to obtain efficiently extremely accurate numerical data. We should point out that the numerical implementation of the determinant formulas derived in this paper can be done in less than ten lines of code in Mathematica or Maple (see [44]) and can be used to obtain very precise results (errors smaller than 10^{-10}). This level of numerical efficiency (the running time on a personal computer is of the order of seconds) and precision cannot be achieved using other methods, such as the ones used in [20–22], which are also inapplicable at finite temperature. In the case of static correlators we also derive the large distance asymptotic behavior and compute the momentum distribution function at zero and finite temperature. Our results show that the Green's function $\mathsf{g}_-^{(T)}(m) = \langle a_{m+1}^\dagger a_1\rangle_T$ satisfies $\mathsf{g}_-^{(T)}(m) = \bar{\mathsf{g}}_-^{(T)}(-m)$ with $\mathrm{Im}\mathsf{g}_-^{(T)}(m) \neq 0$ for intermediate values of the statistics parameter, which explains why the ground-state momentum distribution of anyons is asymmetric [14, 15, 20] with a peak whose location in momentum space depends on the statistics parameter. The location of this peak, which is present even at finite temperature, is extremely important from the experimental point of view due to the fact that the statistics of the particles can be inferred from its position.

The plan of the paper is as follows. In sections 2 and 3 we present the anyonic model and the determinant representations which constitute one of the main results of our paper. Analytical and numerical data for the the asymptotic behaviour and momentum distribution can be found in section 4. The derivation of the determinant representations is presented in section 5. We end with some conclusions in section 6. Some technical details of the calculations can be found in three appendices.

2. The model

We consider a system of one-dimensional hard-core anyons [19–22] in a tight-binding model with the Hamiltonian

$$H = -J\sum_{j=1}^{L}\left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j\right) + 2h\sum_{j=1}^{L} a_j^\dagger a_j\,, \tag{1}$$

where J is the hopping parameter, L is the number of lattice sites which we will consider to be even and $h \in [0, J)$ is the chemical potential. The operators $a_j, a_j^\dagger$ satisfy anyonic commutation relations

$$a_j a_k^\dagger = \delta_{jk} - \mathrm{e}^{-i\pi\kappa\epsilon(j-k)} a_k^\dagger a_j\,, \quad a_j a_k = -\mathrm{e}^{i\pi\kappa\epsilon(j-k)} a_k a_j\,, \quad a_j^\dagger a_k^\dagger = -\mathrm{e}^{\mathrm{i}\pi\kappa\epsilon(j-k)} a_k^\dagger a_j^\dagger, \tag{2}$$

with

$$\epsilon(k) = k/|k|\,, \qquad \epsilon(0) = 0, \tag{3}$$

and $\kappa \in [0, 1]$ the statistics parameter. For $j = k$ we have $a_j^2 = (a_j^\dagger)^2 = 0$ (hard-core condition) and $\{a_j, a_j^\dagger\} = 1$. Varying the statistics parameter κ the commutation relations (2) interpolate between the ones for spinless fermions ($\kappa = 0$) and hard-core bosons ($\kappa = 1$). Some of the ground-state properties and relaxation dynamics of the model were studied in [19–22]. We should point out that the Hamiltonian (1) is a particular case ($U = 0$) of the XXZ spin chain with fractional statistics first considered by Amico, Osterloh and Eckern in [23].

Defining the Fock vacuum in the usual fashion $a_i|0\rangle = 0$, $\langle 0|a_i^\dagger = 0$, $\langle 0|0\rangle = 1$, the eigenstates of the Hamiltonian (1) with N particles are

$$|\Psi_N(\{p\})\rangle = \frac{1}{\sqrt{N!}} \sum_{m_1=1}^{L} \cdots \sum_{m_N=1}^{L} \chi_N(m_1, \cdots, m_N | p_1, \cdots, p_N) a_{m_N}^\dagger \cdots a_{m_1}^\dagger |0\rangle\,, \tag{4}$$

with χ_N the N-body wavefunction and $\{p\}$ the momenta of the particles. Similar to the case of Lieb–Liniger anyons [10] the order in which the creation operators appear in (4) is important as we will see in the subsequent calculation of form factors. A direct consequence of this ordering and commutation relations (2) is that the exchange symmetry of the wavefunction is given by

$$\chi_N(\cdots, m_i, m_{i+1}, \cdots) = -\mathrm{e}^{\mathrm{i}\pi\kappa\epsilon(m_i - m_{i+1})} \chi_N(\cdots, m_{i+1}, m_i, \cdots)\,. \tag{5}$$

The solutions of the Schrödinger equation $-J\sum_{j=1}^{N}[\chi_N(m_1, \cdots, m_i + 1, \cdots, m_N) + \chi_N(m_1, \cdots, m_i - 1, \cdots, m_N)] + 2hN\chi_N(m_1, \cdots, m_N) = E\chi_N(m_1, \cdots, m_N)$ with the appropriate symmetry (5) are

$$\chi_N(m_1, \cdots, m_N | \{p\}) = \frac{\mathrm{i}^{N(N-1)/2}}{\sqrt{N!}} \exp\left(\mathrm{i}\frac{\pi\kappa}{2} \sum_{1\leqslant a<b\leqslant N} \epsilon(m_a - m_b)\right) \times \sum_{P\in S_N} (-1)^{[P]} \left(\prod_{a=1}^{N} \mathrm{e}^{\mathrm{i}m_a p_{P(a)}}\right), \tag{6}$$

with S_N the group of permutations of N elements and we denoted by $(-1)^{[P]}$ the signature of the permutation P. The $\mathrm{i}^{N(N-1)/2}$ factor is added for convenience so that in the bosonic limit ($\kappa = 1$) the wavefunctions reduce to those used in [45] for hard-core bosons (XX0 spin chain). The eigenspectrum of the system is

$$E(\{p\}) = \sum_{j=1}^{N} \varepsilon(p_j), \quad \varepsilon(p) = -2J\cos p + 2h. \tag{7}$$

Due to the exchange symmetry (5), imposing periodic boundary conditions (PBC) in an anyonic system has nontrivial consequences as was first noticed by Averin and Nesteroff [19] (for a detailed discussion see appendix A of [10]). Let us consider the simple case of two particles. If we impose PBC on the first particle $\chi(0,m_2)=\chi(L,m_2)$ using (5) we obtain $\chi(m_2,0)=\mathrm{e}^{2\mathrm{i}\pi\kappa}\chi(m_2,L)$. This shows that if the wavefunction is periodic in the first variable then, due to the anyonic symmetry, the wavefunction will have twisted boundary conditions in the second variable. The generalization in the case of N particles is [10]

$$\begin{aligned}\chi_N(0,m_2,\cdots,m_N) &= \chi_N(L,m_2,\cdots,m_N)\,,\\ \chi_N(m_1,0,m_3,\cdots,m_N) &= \mathrm{e}^{2\pi\mathrm{i}\kappa}\chi_N(m_1,L,m_3,\cdots,m_N)\,,\\ &\vdots\\ \chi_N(m_1,\cdots,m_{N-1},0) &= \mathrm{e}^{2\pi\mathrm{i}\kappa(N-1)}\chi_N(m_1,\cdots,m_{N-1},L)\,.\end{aligned}$$

Imposing PBC on (6) we obtain the Bethe Ansatz Equations (BAEs) satisfied by the momenta

$$\mathrm{e}^{\mathrm{i}p_aL}=\mathrm{e}^{-\mathrm{i}\pi\kappa(N-1)}\,,\quad a=1,\cdots,N\,. \tag{8}$$

The BAEs (8) reproduce the well known equations for hard-core bosons (XX0 spin chain) and spinless fermions for $\kappa=1$ and $\kappa=0$, respectively. Using the commutation relations (2) and the BAEs it is easy to show that the eigenstates (4) satisfy the orthogonality condition $\langle\Psi_{N_1}|\Psi_{N_2}\rangle=0$ if $N_1\neq N_2$ and $\langle\Psi_N(\{p\})|\Psi_N(\{p'\})\rangle=0$ if $\{p\}\neq\{p'\}$. The normalization is[1]

$$\langle\Psi_N(\{p\})|\Psi_N(\{p\})\rangle=L^N\,. \tag{9}$$

Introducing the notation $\{[x]\}=\gamma$, if $x=2\pi\times \mathit{integer}+2\pi\gamma$, $\gamma\in(-1/2,1/2]$, the values of the momenta in the ground state with N particles, N odd, are

$$p_a=\frac{2\pi}{L}\left(-\frac{N+1}{2}+j\right)+\frac{2\pi\delta}{L},\quad \delta=\{[-\pi\kappa(N-1)]\},\quad j=1,\cdots,N. \tag{10}$$

In the thermodynamic limit the momenta fill densely the Fermi zone $[-k_{\mathrm{F}},k_{\mathrm{F}}]$ with $k_{\mathrm{F}}=\arccos(h/J)$ the Fermi momentum. We remind the reader that $h\in[0,J)$ and the system is gapless. For $h>J$ the structure of the ground state is different and will not be considered in this paper. For $h\in[0,J)$ the thermodynamic behavior of hard-core anyons and spinless fermions is the same.

3. Determinant representation for the correlation functions

We are interested in calculating the thermodynamic limit of the time-, space-, and temperature-dependent Green's functions

$$\mathsf{g}_+^{(T)}(m,t)\equiv\frac{\mathsf{tr}\ \left[\mathrm{e}^{-H/T}a_{m_2}(t_2)a_{m_1}^\dagger(t_1)\right]}{\mathsf{tr}\ \left[\mathrm{e}^{-H/T}\right]}\,, \tag{11a}$$

$$\mathsf{g}_-^{(T)}(m,t)\equiv\frac{\mathsf{tr}\ \left[\mathrm{e}^{-H/T}a_{m_2}^\dagger(t_2)a_{m_1}(t_1)\right]}{\mathsf{tr}\ \left[\mathrm{e}^{-H/T}\right]}\,, \tag{11b}$$

where $a_m(t)=\mathrm{e}^{\mathrm{i}Ht}a_m\mathrm{e}^{-\mathrm{i}Ht}$, $a_m^\dagger(t)=\mathrm{e}^{\mathrm{i}Ht}a_m^\dagger\mathrm{e}^{-\mathrm{i}Ht}$ and $m=m_2-m_1\,,\ t=t_2-t_1$. Using the summation of form-factors we were able to express these correlators as Fredholm

[1] More precisely if $\{p'\}=R\{p\}$ where $R\in S_N$ we have $\langle\Psi_N(\{p\})|\Psi_N(\{p'\})\rangle=L^N(-1)^R$.

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determinants which, as we will show in section 4, can be used to obtain extremely precise numerical data. We would like to stress the fact that our result is exact and does not employ any approximations. The derivation of the determinant representation is quite involved; therefore, in this section, we are going to present only the final results. The interested reader can find the full derivation in section 5.

The correlation function $\mathsf{g}_+^{(T)}(m,t)$ has the following representation in terms of Fredholm determinants (for the definition of a Fredholm determinant see section 5.4):

$$\begin{aligned} \mathsf{g}_+^{(T)}(m,t) &= \mathrm{e}^{-2\mathrm{i}ht}\left[G(m,t)+\frac{\partial}{\partial z}\right]\det\left(1+\hat{\mathsf{V}}_T - z\hat{\mathsf{R}}_T^{(+)}\right)\Big|_{z=0}\,, \\ &= \mathrm{e}^{-2\mathrm{i}ht}\left[(G(m,t)-1)\det(1+\hat{\mathsf{V}}_T)+\det(1+\hat{\mathsf{V}}_T-\hat{\mathsf{R}}_T^{(+)})\right], \end{aligned} \tag{12}$$

with $\hat{\mathsf{V}}_T$ and $\hat{\mathsf{R}}_T^{(+)}$ integral operators acting on the interval $[-\pi,\pi]$

$$\left(\hat{\mathsf{V}}_T f\right)(p) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathsf{V}_T(p,p')f(p')\mathrm{d}p', \quad \left(\hat{\mathsf{R}}_T^{(+)} f\right)(p) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathsf{R}_T^{(+)}(p,p')f(p')\mathrm{d}p', \tag{13}$$

with kernels

$$\mathsf{V}_T(p,p') = \sin^2\left(\frac{\pi\kappa}{2}\right)\left[\frac{E_+^T(p)E_-^T(p')-E_-^T(p)E_+^T(p')}{\tan\frac{1}{2}(p-p')} - G(m,t)E_-^T(p)E_-^T(p')\right], \tag{14a}$$

$$\mathsf{R}_T^{(+)}(p,p') = \sin^2\left(\frac{\pi\kappa}{2}\right)E_+^T(p)E_+^T(p'). \tag{14b}$$

The functions appearing in (14) are defined as

$$G(m,t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathrm{d}p\ \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p} \tag{15}$$

$$E_-^T(p) \equiv E_-^T(p,m,t) = \sqrt{\theta(p)}\ \mathrm{e}^{-\mathrm{i}mp/2-\mathrm{i}Jt\cos p}, \tag{16}$$

$$E_+^T(p) \equiv E_+^T(p,m,t) = E^T(p)\,E_-^T(p), \tag{17}$$

$$E^T(p) \equiv E^T(p,m,t) = \mathrm{P.V.}\frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q}}{\tan\frac{1}{2}(q-p)}\mathrm{d}q - \cot\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}. \tag{18}$$

with P.V. denoting the principal value of the integral and $\theta(p)\equiv\theta(p,h,T)$ is the Fermi function

$$\theta(p) = \frac{1}{1+\mathrm{e}^{\frac{-2J\cos p+2h}{T}}}\,. \tag{19}$$

A similar representation is obtained for the $\mathsf{g}_-^{(T)}(m,t)$ correlation function

$$\begin{aligned} \mathsf{g}_-^{(T)}(m,t) &= \mathrm{e}^{2\mathrm{i}ht}\frac{\partial}{\partial z}\det\left(1+\hat{\mathsf{V}}_T + z\hat{\mathsf{R}}_T^{(-)}\right)\Big|_{z=0}\,, \\ &= \mathrm{e}^{2\mathrm{i}ht}\left[\det(1+\hat{\mathsf{V}}_T+\hat{\mathsf{R}}_T^{(-)})-\det(1+\hat{\mathsf{V}}_T)\right]. \end{aligned} \tag{20}$$

In (20), $\hat{\mathsf{V}}_T$ is the same integral operator which appears in the determinant representation of $\mathsf{g}_+^{(T)}(m,t)$. $\hat{\mathsf{R}}_T^{(-)}$ is an integral operators which acts on $[-\pi,\pi]$

$$\left(\hat{\mathsf{R}}_T^{(-)} f\right)(p) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathsf{R}_T^{(-)}(p,p')f(p')\mathrm{d}p'\,,$$

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with kernel

$$\mathsf{R}_T^{(-)}(p,p') = E_-^T(p)E_-^T(p') \,. \tag{21}$$

The main difference in this case is that the $E_+^T(p)$ function which enters the definition of the kernel (14a) is now defined as (note the sign change)

$$E^T(p) \equiv E^T(p,m,t) = \text{P.V.}\frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q}}{\tan\frac{1}{2}(q-p)}\,\mathrm{d}q + \cot\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}. \tag{22}$$

In the $\kappa \to 1$ limit (12) and (20) reproduce the well-known results for hard-core bosons [45] (note that in our notation, $\mathsf{g}_+^{(T)}(m,t)$ and $\mathsf{g}_-^{(T)}(m,t)$ correspond to $\langle\sigma_m^+(t)\sigma_0^-(0)\rangle_T$ and $\langle\sigma_m^-(t)\sigma_0^+(0)\rangle_T$ of [45]). At $\kappa = 0$ we obtain the results for spinless free fermions on the lattice.

3.1. Static limit

Certain simplifications occur in the static limit $t = 0$. Due to the relation

$$\mathsf{g}_+^{(T)}(m,0) = \delta_{m,0} - \mathrm{e}^{-\mathrm{i}\pi\kappa\epsilon(m)}\mathsf{g}_-^{(T)}(-m,0)\,, \tag{23}$$

it will be sufficient to consider only $\mathsf{g}_-^{(T)}(m,0)$. In this limit we have $G(m,0) = \delta_{m,0}$, $E_-^T(p,m,0) = \sqrt{\theta(p)}\mathrm{e}^{-\mathrm{i}mp/2}$, and

$$E_+^T(p,m,0) = \begin{cases} \mathrm{i}\sqrt{\theta(p)}(1-\mathrm{i}\cot(\pi\kappa/2))\mathrm{e}^{\mathrm{i}mp/2}\,, & m>0\,,\\ -\sqrt{\theta(p)}\cot(\pi\kappa/2)\,, & m=0\,,\\ -\mathrm{i}\sqrt{\theta(p)}(1+\mathrm{i}\cot(\pi\kappa/2))\mathrm{e}^{-\mathrm{i}|m|p/2}\,, & m<0\,. \end{cases}$$

The static limit of (12) for $m > 0$ is

$$\mathsf{g}_-^{(T)}(m,0) = \det(1+\hat{\mathsf{v}}_T+\hat{\mathsf{r}}_T^{(-)}) - \det(1+\hat{\mathsf{v}}_T)\,, \tag{24}$$

with $\hat{\mathsf{v}}_T$ and $\hat{\mathsf{r}}_T^{(-)}$ integral operators acting on $[-\pi,\pi]$ like in equation (13) and kernels

$$\begin{aligned} \mathsf{v}_T(p,p') &= (\mathrm{e}^{\mathrm{i}\pi\kappa}-1)\sqrt{\theta(p)}\,\frac{\sin\frac{1}{2}m(p-p')}{\tan\frac{1}{2}(p-p')}\,\sqrt{\theta(p')}\,,\\ \mathsf{r}_T^{(-)}(p,p') &= \sqrt{\theta(p)}\,\mathrm{e}^{-\frac{\mathrm{i}m}{2}(p+p')}\,\sqrt{\theta(p')}\,. \end{aligned} \tag{25}$$

In the case of $m < 0$ the representation (24) remains valid but now the kernels of the integral operators are

$$\begin{aligned} \mathsf{v}_T(p,p') &= (\mathrm{e}^{-\mathrm{i}\pi\kappa}-1)\sqrt{\theta(p)}\,\frac{\sin\frac{1}{2}|m|(p-p')}{\tan\frac{1}{2}(p-p')}\,\sqrt{\theta(p')}\,,\\ \mathsf{r}_T^{(-)}(p,p') &= \sqrt{\theta(p)}\,\mathrm{e}^{\frac{\mathrm{i}|m|}{2}(p+p')}\,\sqrt{\theta(p')}\,. \end{aligned} \tag{26}$$

This shows that for intermediate statistics $\mathrm{Im}\mathsf{g}_-^{(T)}(m,0) \neq 0$ and

$$\mathsf{g}_-^{(T)}(m,0) = \bar{\mathsf{g}}_-^{(T)}(-m,0)\,, \tag{27}$$

with the bar denoting complex conjugation. As we will see in section 4.3 the direct consequence of this relation is that the momentum distribution of anyons is no longer symmetric around the origin.

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3.2. Zero temperature limit

The zero temperature limit results can be derived easily noticing that at $T = 0$ the Fermi function (19) becomes the characteristic function of the interval $[-k_{\mathrm{F}}, k_{\mathrm{F}}]$ with $k_{\mathrm{F}} = \arccos(h/J)$. Therefore, the ground-state Green's functions have the same determinant representations (12), (20), (24) with integral operators acting on $[-k_{\mathrm{F}}, k_{\mathrm{F}}]$

$$\left(\hat{\mathsf{V}}f\right)(p) = \frac{1}{2\pi}\int_{-k_{\mathrm{F}}}^{k_{\mathrm{F}}} \mathsf{V}(p,p')f(p')\mathrm{d}p'\,, \quad \left(\hat{\mathsf{R}}^{(\pm)}f\right)(p) = \frac{1}{2\pi}\int_{-k_{\mathrm{F}}}^{k_{\mathrm{F}}} \mathsf{R}^{(\pm)}(p,p')f(p')\mathrm{d}p',$$

and similar expressions for $\hat{\mathsf{v}}\,, \hat{\mathsf{r}}^{(+)}$ and $E_-^T(p) \to E_-(p) = \mathrm{e}^{-\mathrm{i}mp/2 - \mathrm{i}Jt\cos p}$.

4. Asymptotic behavior of static correlators

Similar Fredholm determinant representations for the correlation functions of other integrable systems were previously obtained in the case of impenetrable bosons [46, 47], XX0 spin chain [45], two-component bosons and fermions [48] and Lieb-Liniger anyons [11]. In all these cases, including ours, the relevant integral operators belong to the so-called 'integrable' class of integral operators [49–51] and as a result the correlation functions satisfy a completely integrable classical system of differential equations. The determinant representation and the associated differential equations represent the basis for the rigorous investigation of the large time and distance asymptotic behavior of the correlators via the solution of a certain matrix Riemann–Hilbert problem. This rather involved program was implemented for the impenetrable Bose gas in [47, 50, 52], for the XX0 spin chain in [53], for the two-component fermions in [54–56] and for Lieb–Linger anyons in [12, 13].

It would be natural to expect a comparable number of papers in the literature devoted to the numerical exploration of the determinant representations, which would allow for a better understanding of the intermediate distance regime which is inaccessible by analytical methods. This is unfortunately not true (two notable exceptions being [58, 59]). While this might be attributed to the possible uncontrollable errors in the evaluation of an infinite determinant, recently Bornemann [44] provided a simple and easily implementable method which allows for very precise numerical evaluations of such representations. This method, which is based on the Nyström solution of the Fredholm integral equations of the second kind with the Gauss-Legendre as the quadrature rule, will be used to calculate the short and intermediate distance static correlation functions and also to check the validity of the large distance asymptotics which we will derive below.

4.1. Asymptotic behavior at zero temperature

At zero temperature we expect the system to be critical and the large distance asymptotic behavior of the correlation functions can be derived using Conformal Field Theory ideas. More precisely we are going to use Cardy's result [60] relating the conformal dimensions of the conformal fields present in the theory from the finite size corrections of the low-lying excitations of the system (a detailed presentation of the method can be found in chap. XVIII of [50]). For our system we have three type of low-lying excitations: addition of one or more particles into the system with quantum number ΔN, backscattering of d particles over the Fermi 'sea' with quantum number d and 'particle-hole' excitations close to the left or right Fermi surface with quantum numbers $N^{\pm}$. The asymptotic behavior of the correlation functions is then

$$\langle\phi(m)\phi(0)\rangle = \sum_{Q} A(Q)\frac{\mathrm{e}^{\mathrm{i}p_Q m}}{|m|^{2\Delta_Q^+ + 2\Delta_Q^-}}\,,$$

where $Q = \{\Delta N, d, N^{\pm}\}$, $A(Q)$ are amplitudes which cannot be obtained using this method, p_Q is the macroscopic part of the momentum gap and the conformal dimensions $\Delta_Q^{\pm}$ can be obtained from the finite size corrections of the energy and momentum using (v_{F} is the Fermi velocity)

$$\begin{aligned} P_Q - P_0 &= p_Q + \frac{2\pi}{L}\left(\Delta_Q^+ - \Delta_Q^-\right)\,, \\ E_Q - E_0 &= \frac{2\pi v_{\mathrm{F}}}{L}\left(\Delta_Q^+ + \Delta_Q^-\right)\,. \end{aligned} \tag{28}$$

The derivation of the finite size corrections in our model is very similar to the one performed for the XX0 spin chain[2] (chap. II of [50]) and can be found in appendix A. The central charge of the model is equal to one and

$$\begin{aligned} P_{\Delta N=1,d,N^{\pm}} - P_0 &= 2k_{\mathrm{F}}\left(d + \frac{1-\kappa}{2}\right) + \frac{2\pi}{L}\left[\left(d + \frac{1-\kappa}{2}\right) + N^+ + N^-\right]\,, \\ E_{\Delta N=1,d,N^{\pm}} - E_0 &= \frac{2\pi v_{\mathrm{F}}}{L}\left[\left(\frac{1}{2}\right)^2 + \left(d + \frac{1-\kappa}{2}\right)^2 + N^+ + N^-\right]. \end{aligned} \tag{29a}$$

Neglecting the contributions coming from the $N^{\pm}$ terms we obtain the following asymptotic expansion for the Green's function

$$\mathsf{g}_-^{(0)}(m) \sim \sum_{d\in\mathbb{Z}} A(d)\frac{\mathrm{e}^{\mathrm{i}k_{\mathrm{F}}[2d+(1-\kappa)]m}}{|m|^{\frac{[2d+(1-\kappa)]^2}{2}+\frac{1}{2}}}\,, \tag{30}$$

where d is the number of backscattered particles. For $\kappa = 1$ this expansion reduces to the Green's function asymptotic expansion of hard-core bosons with leading term $\mathsf{g}_-^{(0)}(m) \sim 1/|m|^{1/2}$. At the free-fermionic point, $\kappa = 0$, neglecting all the terms except $d = 0, -1$ we get $\mathsf{g}_-^{(0)}(m) \sim \sin(k_{\mathrm{F}}m)/m$ which is in fact the exact result modulo the amplitude. For intermediate values of the statistics parameter the leading term of the expansion, given by $d = 0$, is oscillating with a wavevector proportional to $1-\kappa$. This is a general characteristic of 1D anyonic systems first observed by Calabrese and Mintchev [9] and it can be seen in figure 1. The asymptotic behavior plotted is

$$\tilde{\mathsf{g}}_-^{(0)}(m) = a\,\frac{\mathrm{e}^{\mathrm{i}[(k_{\mathrm{F}}(1-\kappa)m-b]}}{|m|^{\frac{(1-\kappa)^2}{2}+\frac{1}{2}}} + c\,\frac{\mathrm{e}^{\mathrm{i}[k_{\mathrm{F}}(-2+(1-\kappa))m+d]}}{|m|^{\frac{[-2+(1-\kappa)]^2}{2}+\frac{1}{2}}}, \tag{31}$$

with a, b, c, d real parameters obtained using a fitting procedure and the relative errors defined in the usual fashion $\Delta\mathrm{Re}\mathsf{g}_-^{(0)}(m) = |\mathrm{Re}\mathsf{g}_-^{(0)}(m) - \mathrm{Re}\tilde{\mathsf{g}}_-^{(0)}(m)|/|\mathrm{Re}\mathsf{g}_-^{(0)}(m)|$ and a similar expression for the imaginary part. The correlation function $\mathsf{g}_-^{(0)}(m)$ was computed from the zero temperature limit of equation (24). Using the method presented in [44] for

[2] Reference [50] considers the more general case of the XXZ spin chain. The results relevant for us are obtained considering $\Delta = 0$ and the dressed charge $\mathcal{Z} = 1$.

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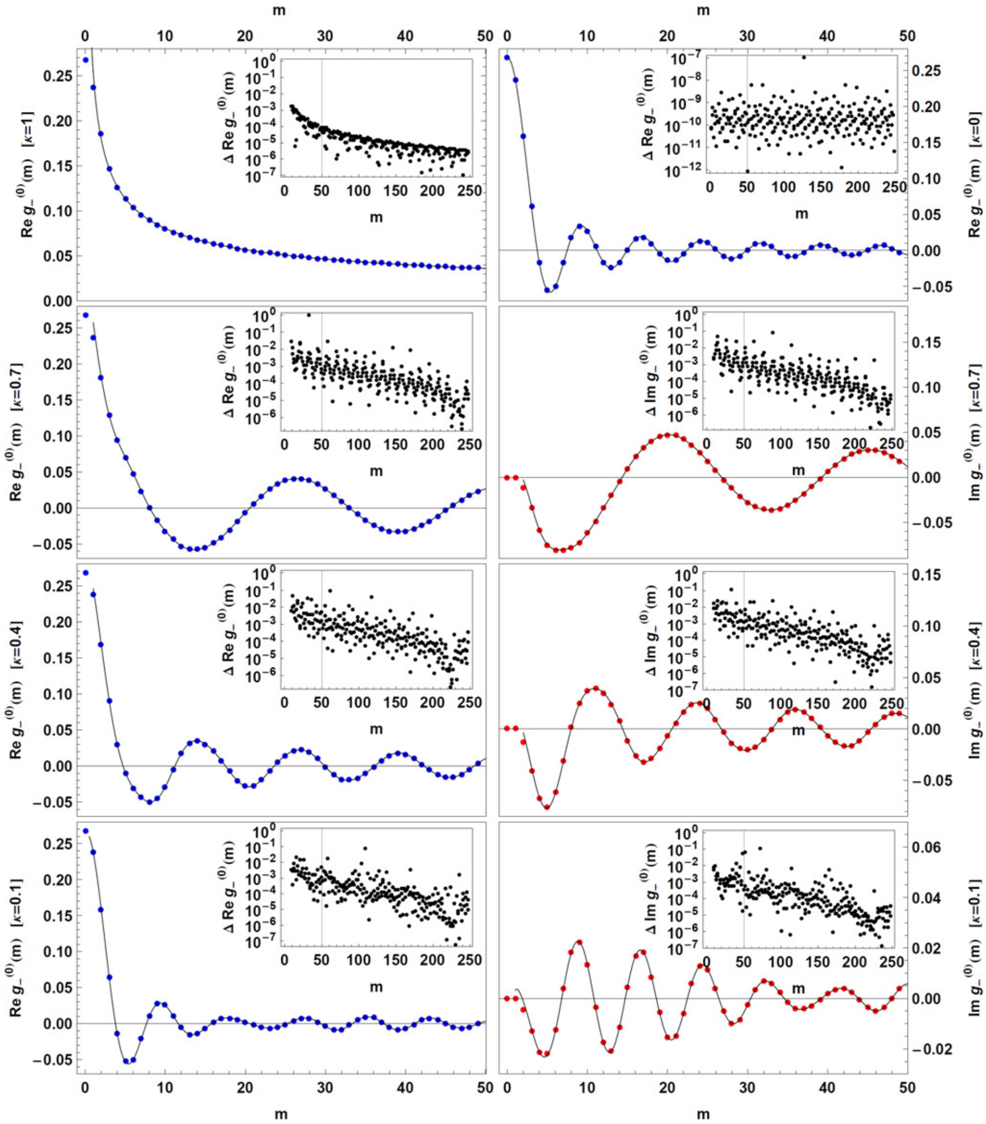

Figure 1. Plot of the real and imaginary parts of the zero temperature correlation function $\mathsf{g}_{-}^{(0)}(m)$ (blue and red dots) and asymptotic behavior (thin line) given by equation (31) for $h = 4/3$, $J = 2$ and different values of the statistics parameter. The results for hard-core bosons ($\kappa = 1$) and spinless free fermions ($\kappa = 0$) are presented in the top panels. The insets contain the relative errors of the asymptotic formula for m up to 250 (the errors of the data presented in the main panels are shown up to the vertical line at $m = 50$). (Distance m in units of a_0 the lattice constant which, for convenience, is set to 1.)

the numerical implementation of Fredholm determinants we were able to obtain extremely accurate values (absolute errors smaller than 10^{-10}).

Another interesting feature specific to correlation functions of 1D anyonic systems (see [9]) is the presence of fermionic beats for values of the statistics parameter close to 0. In this region we can see from the r.h.s. of (31) that we have two oscillations with almost

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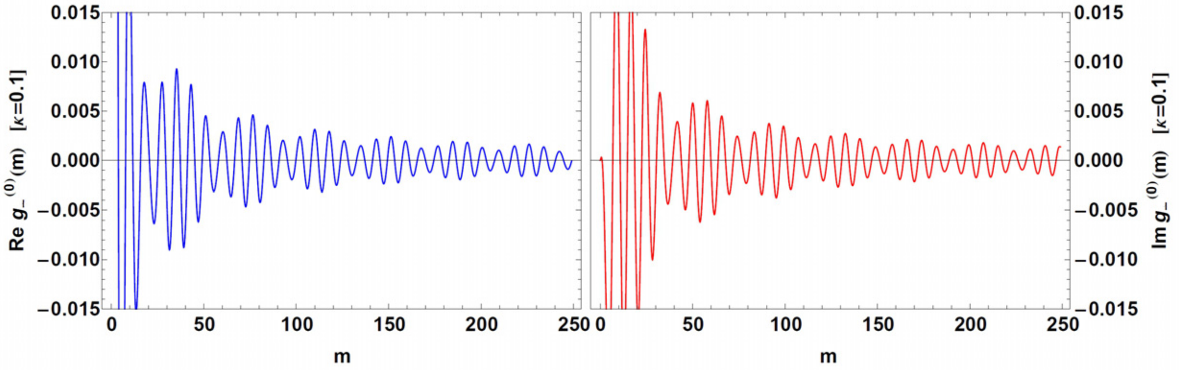

Figure 2. Real and imaginary parts of the correlation function $g_-^{(0)}(m)$ for $h = 4/3$, $J = 2$ and $\kappa = 0.1$ showing fermionic beats. (Distance m in units of a_0.)

equal amplitudes and wavevectors producing beating effects. This phenomenon can be seen in figure 2 for $\kappa = 0.1$.

4.2. Asymptotic behavior at finite temperature

At low temperatures the asymptotic behavior of $g_-^{(T)}(m)$ can be derived from the zero temperature result (30) by replacing $1/|m|$ with $1/\sinh(\pi v_F m/T)$ with v_F the Fermi velocity. However, at higher temperatures the CFT description is no longer valid and we need to use a different method. Here, we present an heuristic derivation of the temperature dependent asymptotic expansion of the Green's function which is based on the similar results obtained rigorously for impenetrable Lieb–Liniger anyons [12]. Even though the considerations below are nothing more than an educated guess, the numerical data presented in figure 3 show that our result, equation (35), is nevertheless correct. In the case of Lieb–Liniger impenetrable anyons, the main term responsible for the exponential decay of the correlators, denoted by $C(\beta, \kappa)$ in equation 3 of [12], contained the logarithm of the ratio between the Fermi distribution and an 'anyonic' distribution function interpolating between the Fermi and Bose distributions. We will assume that for our model the lattice equivalent of this anyonic distribution function is given by

$$\theta(p,\kappa) = \frac{1}{e^{\frac{-2J\cos p + 2h}{T}} - e^{i\pi(1-\kappa)}}, \tag{32}$$

in terms of which we can define the lattice analog of $C(\beta, \kappa)$

$$C(\kappa) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \log\left(\frac{\theta(p,\kappa)}{\theta(p,0)}\right) dp\,. \tag{33}$$

An important role is played by the zeroes of $\theta(p,\kappa)^{-1}$ closest to the real axis and situated in the upper half plane denoted by

$$\lambda_0(\kappa) = \arccos\left[\frac{1}{2J}(2h - i(1-\kappa)\pi T)\right],$$
$$\lambda_{-1}(\kappa) = -\arccos\left[\frac{1}{2J}(2h + i(2\pi T - (1-\kappa)\pi T))\right]. \tag{34}$$

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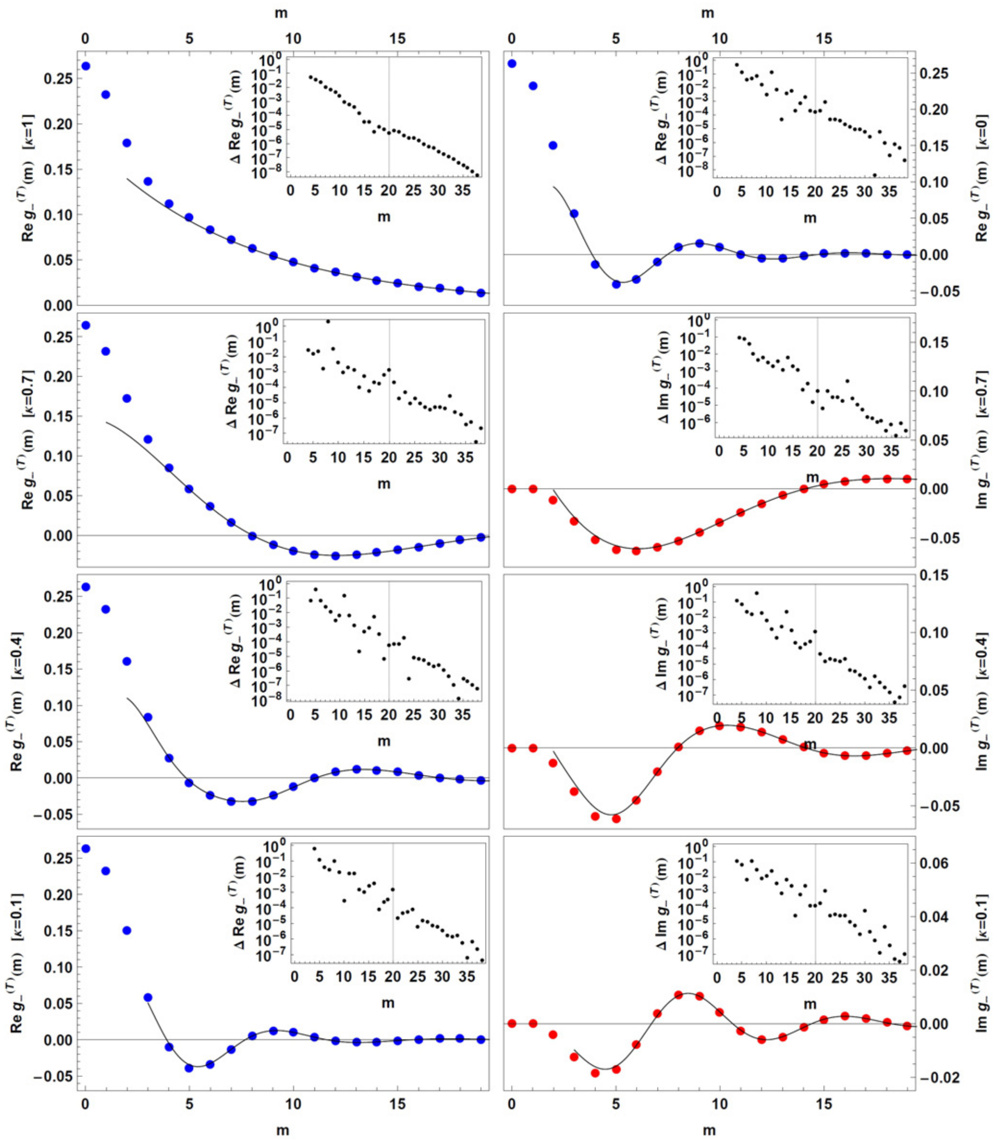

Figure 3. Plot of the real and imaginary parts of the temperature dependent correlation function $\mathrm{g}_-^{(T)}(m)$ (blue and red dots) and asymptotic behavior (thin line) given by equation (35) for $h = 4/3\,, J = 2$ temperature $T = 1/4$ and different values of the statistics parameter. The results for hard-core bosons ($\kappa = 1$) and spinless free fermions ($\kappa = 0$) are presented in the top panels. The insets contain the relative errors of the asymptotic formula for m up to 40 (the errors of the data presented in the main panels are shown up to the vertical line at $m = 20$). The insets contain the relative errors of the asymptotic formula. (Distance m in units of a_0.)

Then, the asymptotic behavior of the correlation function obtained in direct analogy with the continuum result [12] is given by[3]

$$\tilde{\mathrm{g}}_-^{(T)}(m) \simeq (a + \mathrm{i}b)\,\mathrm{e}^{-m[C(\kappa)-\mathrm{i}\lambda_0(\kappa)]} + (c + \mathrm{i}d)\,\mathrm{e}^{-m[C(\kappa)-\mathrm{i}\lambda_{-1}(\kappa)]}, \qquad m > 0, \tag{35}$$

[3] There is a typo in equation (6) of [12]. The correct version is $\lambda_j = (-1)^j \left(\beta + \sqrt{\beta^2 + \pi^2[\kappa + 2j]^2}\right)^{1/2} / \sqrt{2} + \cdots$.

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with a, b, c, d real parameters. For $\kappa = 1$ the second term in the r.h.s. of equation (35) is much smaller than the first term and we obtain the known result for hard-core bosons $\tilde{\mathrm{g}}_-^{(T)}(m, \kappa = 1) \simeq a\, \mathrm{e}^{\frac{m}{2\pi}\int_{-\pi}^{+\pi}\log\left|\tanh\left(\frac{h - J\cos p}{T}\right)\right|\mathrm{d}p}$ [53]. In the fermionic limit $C(\kappa = 0) = 0$ and we have $\tilde{\mathrm{g}}_-^{(T)}(m, \kappa = 0) \simeq a\, \mathrm{e}^{\mathrm{i}m\lambda_0(0)} + c\, \mathrm{e}^{\mathrm{i}m\lambda_{-1}(0)}$ with $\lambda_j = (-1)^j \arccos\left[\frac{1}{2J}(2h - (-1)^j \mathrm{i}\pi T)\right]$, $j = 0, -1$. This result reproduces the first two terms of the asymptotic expansion of free fermions which can also be derived starting from $\mathrm{g}_-^{(T)}(m, \kappa = 0) \simeq \int_{-\pi}^{+\pi} \mathrm{e}^{\mathrm{i}pm}\theta(p)\,\mathrm{d}p$ and moving the integration contour in the upper half-plane for $m > 0$. Plots of the correlation function $\mathrm{g}_-^{(T)}(m)$ and the asymptotic expansion equation (35) for $\kappa = 0, 1$ are shown in the top panels of figure 3. Additional numerical checks of our asymptotic expansion for intermediate values of the statistics parameter are presented in the lower panels of figure 3. For values of κ in the interval [0.4,1] it is sufficient to fit the data using only the first term in the r.h.s. of (35), obtaining extremely accurate results (relative errors of 10^{-8} for $m = 40$ and $\kappa = 0.4$). As κ approaches the fermionic point, $\kappa = 0$, we need to use both terms in order to obtain accurate results (relative errors of 10^{-8} for $m = 40$ and $\kappa = 0.1$). The oscillatory behavior of the correlation functions is still present but at finite temperature the damping is exponential compared to the algebraic decay at zero temperature. The rate of exponential decay is a decreasing function of κ, being maximum for spinless fermions.

4.3. Momentum distribution at zero and finite temperature

The accurate knowledge of the large distance asymptotic behavior allows us to compute the momentum distribution function defined as

$$n(k) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}km}\, \mathrm{g}_-(m)\,. \tag{36}$$

A direct consequence of equation (27) is that $\mathrm{Re}\,\mathrm{g}_-(-m) = \mathrm{Re}\,\mathrm{g}_-(m)$ and $\mathrm{Im}\,\mathrm{g}_-(-m) = -\mathrm{Im}\,\mathrm{g}_-(m)$ which means that

$$n(k) = \frac{1}{\pi}\left[\frac{\mathrm{Re}\,\mathrm{g}_-(0)}{2} + \sum_{m=1}^{\infty}\mathrm{Re}\,\mathrm{g}_-(m)\cos(km) - \sum_{m=1}^{\infty}\mathrm{Im}\,\mathrm{g}_-(m)\sin(km)\right]. \tag{37}$$

For hard-core bosons and spinless fermions $\mathrm{Im}\,\mathrm{g}_-(m) = 0$ and, therefore, the momentum distribution is symmetric with respect to the k axis. However, in the case of anyonic systems $\mathrm{Im}\,\mathrm{g}_-(m) \neq 0$ and the momentum distribution is asymmetric (see also [14–16]). Also, from the asymptotic behavior (31) we expect that at zero temperature the momentum distribution will have a singularity at $k = (1-\kappa)k_\mathrm{F}$ and a weaker singularity at $[-2 + (1-\kappa)]k_\mathrm{F}$. The numerical results presented in figure 4 confirm these theoretical predictions. At zero temperature we can see clearly how the peak present at $k = 0$ for $\kappa = 1$ (hard-core bosons) decreases and moves to the right as $(1-\kappa)k_\mathrm{F}$ and becomes the discontinuity at k_F of the momentum distributions for spinless fermions. The weak singularity at $-2k_\mathrm{F}$ for $\kappa = 1$ manifests itself in the derivative of $n(k)$ which becomes sharper with decreasing κ and becomes the discontinuity of the momentum distribution at $-k_\mathrm{F}$ (see also [14]). At finite temperature the momentum distribution gets smoother and wider. Even so, the peaks at $(1-\kappa)k_\mathrm{F}$ still remain visible and can be experimentally detected.

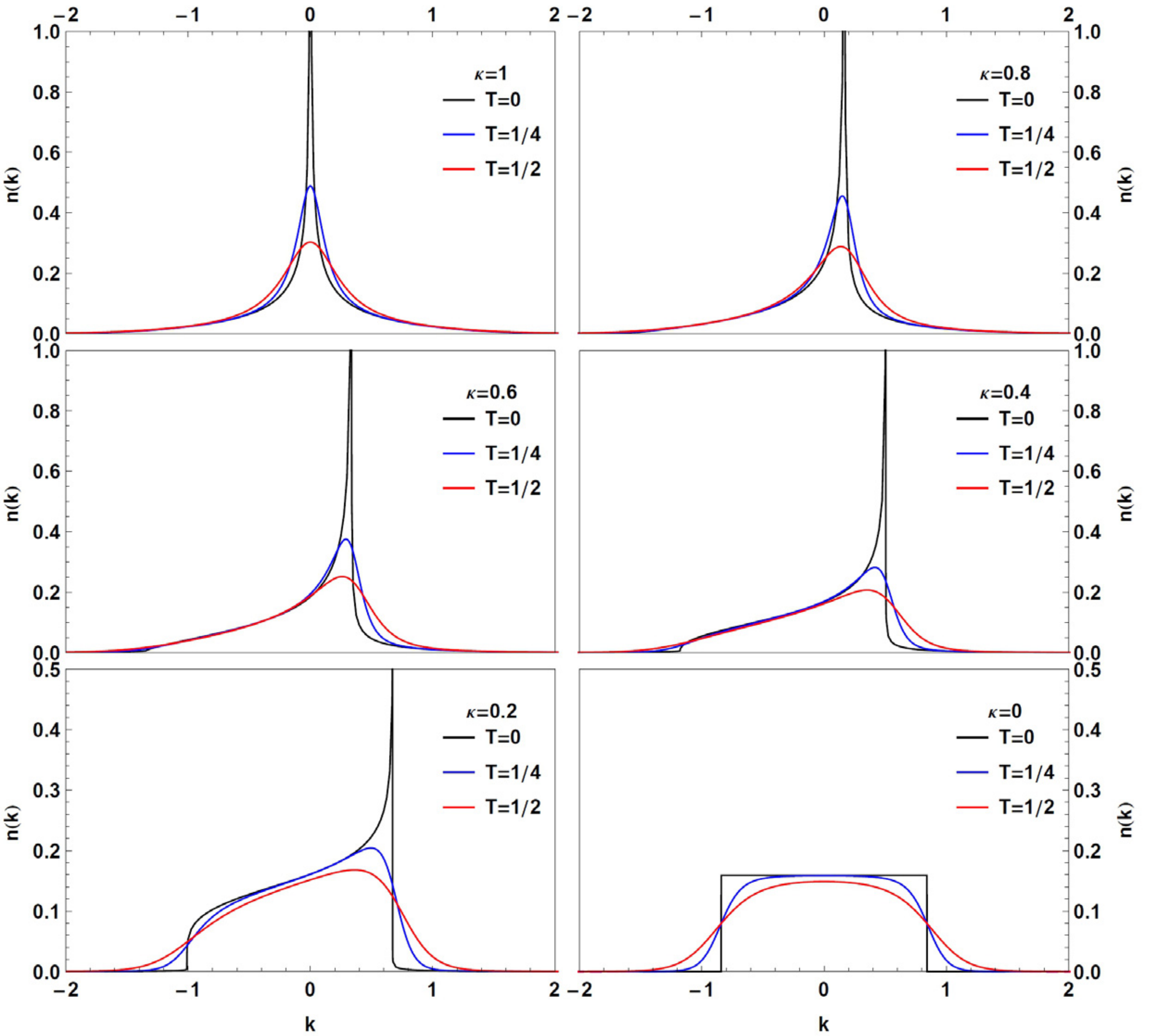

Figure 4. Momentum distribution functions for $h = 4/3\,, J = 2$ temperature $T = 0\,, 1/4\,, 1/2$ and different values of the statistics parameter. (Momentum k in units of $1/a_0$.)

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5. Derivation of the determinant representation for the correlation functions

The determinant representations presented in section 3 were derived using the method known as the summation of form-factors [11, 47, 48, 50], which is briefly sketched below (an alternative method based on Wick's theorem can be found in [57]). We start from the finite lattice expression of the Green's function (we focus on $\mathsf{g}_+^{(T)}(m,t)$, the $\mathsf{g}_-^{(T)}(m,t)$ case can be treated along similar lines)

$$\mathsf{g}_+^{(T)}(m,t) = \frac{1}{\mathsf{tr}\ [\mathrm{e}^{-H/T}]} \sum_{N=0}^{L} \sum_{p_1<\cdots<p_N} \mathrm{e}^{-\frac{E_N(\{p\})}{T}} \langle a_{m_2}(t_2) a_{m_1}^\dagger(t_1)\rangle_N \,, \tag{38}$$

where we have used the short notation $\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \langle\Psi_N(\{p\})|a_{m_2}(t_2)a^\dagger_{m_1}(t_1)|\Psi_N(\{p\})\rangle/\langle\Psi_N(\{p\})|\Psi_N(\{p\})\rangle$. Inserting a resolution of identity we obtain

$$\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \sum_{\{q\}_{N+1}} \frac{\langle\Psi_N(\{p\})|a_{m_2}(t_2)|\Psi_{N+1}(\{q\})\rangle\langle\Psi_{N+1}(\{q\})|a^\dagger_{m_1}(t_1)|\Psi_N(\{p\})\rangle}{\langle\Psi_N(\{p\})|\Psi_N(\{p\})\rangle\langle\Psi_{N+1}(\{q\})|\langle\Psi_{N+1}(\{q\})\rangle}\,,$$

$$= \sum_{\{q\}_{N+1}} \frac{\overline{\mathsf{F}}_N(m_2,t_2,\{q\},\{p\})\mathsf{F}_N(m_1,t_1,\{q\},\{p\})}{L^{2N+1}}\,, \tag{39}$$

where the sum is over all sets of allowed values for the momenta $\{q\}$ with $\dim\{q\} = N+1$, and we have introduced the form factors (note that $\langle\Psi_N(\{p\})|a_m(t)|\Psi_{N+1}(\{q\}\rangle = \overline{\mathsf{F}}_N(m,t,\{q\},\{p\})$)

$$\mathsf{F}_N(m,t,\{q\},\{p\}) \equiv \langle\Psi_{N+1}(\{q\})|a^\dagger_m(t)|\Psi_N(\{p\})\rangle\,. \tag{40}$$

Even though the summation appearing in equation (39) seems daunting, as we will show in the next sections, it can be done exactly in the form of a finite size determinant. Using this representation in equation (38) the thermodynamic limit can be performed explicitly, obtaining the representations given in section 3. Summarizing, the derivation of the Fredholm determinant representation involves three steps: (a) calculation of the form factors on the finite lattice, (b) derivation of a determinant formula for the normalized mean value of bilocal operators via summation of the form factors, and (c) taking the thermodynamic limit.

5.1. Form factors on the finite lattice

The implementation of the program sketched above starts with the calculation of the form factors. Using $a^\dagger_m(t) = \mathrm{e}^{\mathrm{i}Ht}a^\dagger_m\mathrm{e}^{-\mathrm{i}Ht}$ we have

$$\mathsf{F}_N(m,t,\{q\},\{p\}) = \mathrm{e}^{\mathrm{i}t\left(\sum_{a=1}^{N+1}\varepsilon(q_a)-\sum_{b=1}^{N}\varepsilon(p_b)\right)}\mathsf{F}_N(m,\{q\},\{p\}), \tag{41}$$

where we have introduced the static form-factor $\mathsf{F}_N(m,\{q\},\{p\}) = \langle\Psi_{N+1}(\{q\})|a^\dagger_m|\Psi_N(\{p\}\rangle$, which in terms of the wavefunctions can be written as (see appendix B)

$$\mathsf{F}_N(m,\{q\},\{p\}) = \sqrt{N+1}\sum_{m_1=1}^{L}\cdots\sum_{m_N=1}^{L}\overline{\chi}_{N+1}(m_1,\cdots,m_N,m)|\{q\})\chi_N(m_1,\cdots,m_N|\{p\}). \tag{42}$$

In equation (42) $\{q\} = q_1,\cdots,q_{N+1}$, $\mathrm{card}\{q\} = N+1$ and $\{p\} = q_1,\cdots,q_N$, $\mathrm{card}\{p\} = N$ are the set of momenta that parameterize the eigenvectors $|\Psi_{N+1}\rangle$ and $|\Psi_N\rangle$. They satisfy the BAEs (8)

$$\mathrm{e}^{\mathrm{i}p_aL} = \mathrm{e}^{-\mathrm{i}\pi\kappa(N-1)}\,,\qquad a = 1,\cdots,N\,,$$
$$\mathrm{e}^{\mathrm{i}q_bL} = \mathrm{e}^{-\mathrm{i}\pi\kappa N}\,,\qquad b = 1,\cdots,N+1\,,$$

and the allowed values are

$$p_j = \frac{2\pi}{L}\left(-\frac{L}{2}+j\right)+\frac{2\pi\delta}{L},\quad \delta = \{[-\pi\kappa(N-1)]\},\quad j = 1,\cdots,L \tag{43a}$$

$$q_l = \frac{2\pi}{L}\left(-\frac{L}{2}+l\right)+\frac{2\pi\delta'}{L},\quad \delta' = \{[-\pi\kappa N]\},\qquad l = 1,\cdots,L. \tag{43b}$$

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Two consequences of (43) which will play an important role in the following are that $q_b - p_a = \frac{2\pi}{L}(l - \kappa/2)$ with l integer and $q_b \neq p_a$ except in the case of spinless fermions ($\kappa = 0$). Using the explicit expressions of the wavefunctions equation (42) can be rewritten as

$$\begin{aligned}
\mathsf{F}_N(m,\{q\},\{p\}) &= \frac{(-\mathrm{i})^N}{N!}\sum_{m_1,\cdots,m_N=1}^{L}\sum_{Q\in S_{N+1}}\sum_{P\in S_N}(-1)^{[Q]+[P]}\mathrm{e}^{-\mathrm{i}mq_{Q(N+1)}} \\
&\quad\times\prod_{a=1}^{N}\left(\mathrm{e}^{-\mathrm{i}m_a(q_{Q(a)}-p_{P(a)})-\mathrm{i}\pi\kappa\epsilon(m_a-m)/2},\right) \\
&= \frac{1}{N!}\sum_{Q\in S_{N+1}}\sum_{P\in S_N}(-1)^{[Q]+[P]}\mathrm{e}^{-\mathrm{i}mq_{Q(N+1)}} \\
&\quad\times\prod_{a=1}^{N}\left((-i)\sum_{m_a=1}^{L}\mathrm{e}^{-\mathrm{i}m_a(q_{Q(a)}-p_{P(a)})-\mathrm{i}\pi\kappa\epsilon(m_a-m)/2}\right).
\end{aligned}$$

The summation over m_i's can be done using $\mathrm{e}^{-\mathrm{i}\pi\kappa\epsilon(n-m)/2} = \cos(\pi\kappa/2) - \mathrm{i}\sin(\pi\kappa/2)\epsilon(n-m)$ and the BAEs, $\mathrm{e}^{\mathrm{i}(q_a-p_b)L} = \mathrm{e}^{-\mathrm{i}\pi\kappa}$, with the result

$$-\mathrm{i}\sum_{n=1}^{L}\mathrm{e}^{-\mathrm{i}n(q_a-p_b)-\mathrm{i}\pi\kappa\epsilon(n-m)/2} = \mathrm{i}\sin\left(\frac{\pi\kappa}{2}\right)\cot\frac{1}{2}(q_a-p_b)\mathrm{e}^{-\mathrm{i}m(q_a-p_b)}.$$

Therefore,

$$\begin{aligned}
\mathsf{F}_N(m,\{q\},\{p\}) &= \frac{(\mathrm{i}\sin(\pi\kappa/2))^N}{N!}\mathrm{e}^{-\mathrm{i}m(\sum_{a=1}^{N+1}q_a-\sum_{b=1}^{N}p_b)}\sum_{P\in S_N}(-1)^{[P]} \\
&\quad\times\begin{vmatrix}\cot\frac{1}{2}(q_1-p_{P(1)}) & \cdots & \cot\frac{1}{2}(q_1-p_{P(N)}) & 1\\ \vdots & \vdots & \vdots & \vdots\\ \cot\frac{1}{2}(q_{N+1}-p_{P(1)}) & \cdots & \cot\frac{1}{2}(q_{N+1}-p_{P(N)}) & 1\end{vmatrix}.
\end{aligned}$$

It is easy to see now that the sum over the permutations P gives $N!$ identical terms because for each P we can permute the first N columns of the determinant with P^{-1} which multiplies the determinant with $(-1)^{[P^{-1}]} = (-1)^{[P]}$ canceling the signature of the permutation. Using this observation together with (41) we obtain the final expression for the form-factor

$$\begin{aligned}
\mathsf{F}_N(m,t,\{q\},\{p\}) &= (\mathrm{i}\sin(\pi\kappa/2))^N\,\mathrm{e}^{\mathrm{i}t\left(\sum_{a=1}^{N+1}\varepsilon(q_a)-\sum_{b=1}^{N}\varepsilon(p_b)\right)-\mathrm{i}m\left(\sum_{a=1}^{N+1}q_a-\sum_{b=1}^{N}p_b\right)} \\
&\quad\times\det{}_{N+1}\mathsf{B},
\end{aligned}\tag{44}$$

where the $(N+1)\times(N+1)$ matrix B has the elements

$$\mathsf{B}_{ab} = \begin{cases}\cot\frac{1}{2}(q_a-p_b) & a=1,\cdots,N+1,\quad b=1,\cdots,N,\\ 1 & a=1,\cdots,N+1,\quad b=N+1.\end{cases}\tag{45}$$

For $\kappa = 1$ the expression for the form factors (44) reduces to the one obtained by the authors of [45] in their study of the XX0 spin chain. Subtracting the last row of B from all the other rows, an operation which leaves the determinant unchanged, we obtain $\det_{N+1}\mathsf{B} = \det_N(\mathsf{A}^{(1)} - z\mathsf{A}^{(2)})|_{z=1}$, with $\mathsf{A}^{(1)}_{ab} = \cot\frac{1}{2}(q_a - p_b)$, $a,b = 1,\cdots,N$, and $\mathsf{A}^{(2)}_{ab} = \cot\frac{1}{2}(q_{N+1} - p_b)$, $a,b = 1,\cdots,N$. Another useful expression, $\det_{N+1}\mathsf{B} = \left(1 + \frac{\mathrm{d}}{\mathrm{d}z}\right)\det_N\mathsf{A}|_{z=0}$, can be derived using the fact that $\det_N\mathsf{A} = \det_N(\mathsf{A}^{(1)} - z\mathsf{A}^{(2)})$ is a linear function in z (the rank of $\mathsf{A}^{(2)}$ is 1) and the identity $\left(1 + \frac{\mathrm{d}}{\mathrm{d}z}\right)f(z)|_{z=0} = f(1)$ valid for a linear function $f(z) = a + bz$.

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5.2. Normalized mean value of $\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N$

Having derived a compact expression for the form factors now we can compute normalized mean values of bilocal operators. We will consider first $\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N$. Starting from equation (39) and using (44) we find

$$\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \frac{(\sin(\pi\kappa/2))^{2N}}{L^{2N+1}} \sum_{q_1<\cdots<q_{N+1}} \mathrm{e}^{-\mathrm{i}t\left(\sum_{a=1}^{N+1}\varepsilon(q_a)-\sum_{b=1}^{N}\varepsilon(p_b)\right)+\mathrm{i}m\left(\sum_{a=1}^{N+1}q_a-\sum_{b=1}^{N}p_b\right)} \times (\det{}_{N+1}\mathsf{B})^2 . \tag{46}$$

In the following it will be useful to introduce the function

$$e_-(m,t,p) \equiv e_-(p) = \mathrm{e}^{-\mathrm{i}mp/2-\mathrm{i}Jt\cos p}\,, \tag{47}$$

in terms of which the exponential appearing in the r.h.s of (46) denoted by $f(m,t,\{q\},\{p\}) \equiv f(\{q\},\{p\})$ is given by

$$f(\{q\},\{p\}) = \mathrm{e}^{-2\mathrm{i}ht}\prod_{a=1}^{N+1}(e_-(m,t,q_a))^{-2}\prod_{b=1}^{N}(e_-(m,t,p_b))^2 .$$

An important observation that we make is that $(\det_{N+1}\mathsf{B})^2$ is a symmetric function of q's which vanishes when two q's coincide. This means that we can write

$$\sum_{q_1<\cdots<q_{N+1}} = \frac{1}{(N+1)!}\sum_{q_1}\cdots\sum_{q_{N+1}}, \tag{48}$$

where the summation over q for an arbitrary function ϕ is given by $\sum_{q_a}\phi(q_a) = \sum_{l=1}^{L}\phi([q_a]_l)$, with $[q_a]_l = \frac{2\pi}{L}\left(-\frac{L}{2}+l\right)+\frac{2\pi\delta'}{L}$, $l=1,\cdots,L$. Using the definition of the determinant $\det_N\mathsf{B} = \sum_{Q\in S_N}(-1)^{[Q]}\prod_{a=1}^{N}\cot\frac{1}{2}(q_{Q(a)}-p_a)$ and (48) we find

$$\begin{aligned}
\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N
&= \frac{(\sin(\pi\kappa/2))^{2N}}{(N+1)!\,L^{2N+1}}\sum_{q_1,\cdots,q_{N+1}} f(\{q\},\{p\})\sum_{P,Q\in S_{N+1}}(-1)^{[P]+[Q]}\\
&\quad\times\prod_{a=1}^{N}\cot\frac{1}{2}(q_{P(a)}-p_a)\cot\frac{1}{2}(q_{Q(a)}-p_a)\\
&= \frac{(\sin(\pi\kappa/2))^{2N}}{(N+1)!\,L^{2N+1}}\sum_{q_1,\cdots,q_{N+1}} f(\{q\},\{p\})\sum_{Q,R\in S_{N+1}}(-1)^{[R]+[Q]+[Q]}\\
&\quad\times\prod_{a=1}^{N}\cot\frac{1}{2}(q_{RQ(a)}-p_a)\cot\frac{1}{2}(q_{Q(a)}-p_a)\\
&= \frac{(\sin(\pi\kappa/2))^{2N}}{(N+1)!\,L^{2N+1}}\sum_{q_1,\cdots,q_{N+1}} f(\{q\},\{p\})\\
&\quad\times\sum_{Q\in S_{N+1}}\begin{vmatrix}\cot\frac{1}{2}(q_{Q(1)}-p_1) & \cdots & \cot\frac{1}{2}(q_{Q(1)}-p_N) & 1\\ \vdots & \vdots & \vdots & \vdots\\ \cot\frac{1}{2}(q_{Q(N+1)}-p_1) & \cdots & \cot\frac{1}{2}(q_{Q(N+1)}-p_N) & 1\end{vmatrix}\\
&\quad\times\prod_{a=1}^{N}\cot\frac{1}{2}(q_{Q(a)}-p_a),
\end{aligned} \tag{49}$$

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where in the second line we have used the fact that for a given permutation Q every permutation P can be written as $P = RQ$. Multiplying the i-th row (column) $(i = 1, \cdots, N)$ of the the determinant appearing in (49) with $e_-(p_i)\cot\frac{1}{2}(q_{Q(i)} - p_i)/e_-^2(q_{Q(i)})$ $(e_-(p_i)\sin^2(\pi\kappa/2)/L^2)$ and the $N+1$-th row (column) with $1/e_-^2(q_{Q_{N+1}})$ $(1/L)$ then q_{Q_i} appears only in the i-th row which means that we can sum over q's inside the determinant. Also the sum over permutation gives $(N+1)!$ identical terms. Introducing the functions

$$\mathsf{U}_{ab} = \frac{\sin^2(\pi\kappa/2)}{L^2}\sum_q \mathrm{e}^{imq+2\mathrm{i}Jt\cos q}\cot\frac{1}{2}(q-p_a)\cot\frac{1}{2}(q-p_b)\,, \tag{50a}$$

$$g(m,t) = \frac{1}{L}\sum_q \mathrm{e}^{imq+2\mathrm{i}Jt\cos q}\,, \tag{50b}$$

$$e(m,t,p) \equiv e(m) = \frac{1}{L}\sum_q \mathrm{e}^{imq+2\mathrm{i}Jt\cos q}\cot\frac{1}{2}(q-p)\,, \tag{50c}$$

$$e_+(m,t,p) \equiv e_+(m) = e_-(m,t,p)e(m,t,p), \tag{50d}$$

we find

$$\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \mathrm{e}^{-2iht}\begin{vmatrix} \mathsf{U}_{11}e_-(p_1)e_-(p_1) & \cdots & \mathsf{U}_{1N}e_-(p_1)e_-(p_N) & e_+(p_1) \\ \vdots & \vdots & \vdots & \vdots \\ \mathsf{U}_{1N}e_-(p_N)e_-(p_1) & \cdots & \mathsf{U}_{NN}e_-(p_N)e_-(p_N) & e_+(p_N) \\ e_+(p_1)\sin^2(\pi\kappa/2)/L & \cdots & e_+(p_N)\sin^2(\pi\kappa/2)/L & g(m,t) \end{vmatrix}. \tag{51}$$

Expanding on the last column we obtain

$$\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \mathrm{e}^{-2iht}\left[g(m,t)+\frac{\mathrm{d}}{\mathrm{d}z}\right]\det{}_N\left(\mathsf{U}_{ab}e_-(p_a)e_-(p_b) - z\mathsf{R}^{(+)}_{ab}\right)\Big|_{z=0},$$

where we have introduced

$$\mathsf{R}^{(+)}_{ab} = \sin^2(\pi\kappa/2)e_+(p_a)e_+(p_b)/L. \tag{52}$$

Making use of the identity $\cot\frac{1}{2}(q-p_a)\cot\frac{1}{2}(q-p_b) = \cot\frac{1}{2}(p_a-p_b)[\cot\frac{1}{2}(q-p_a) - \cot\frac{1}{2}(q-p_b)] - 1$ and introducing

$$d(m,t,p,\kappa) \equiv d(p) = \frac{\sin^2(\pi\kappa/2)}{L^2}\sum_q \frac{\mathrm{e}^{imq+2\mathrm{i}Jt\cos q}}{\sin^2\frac{1}{2}(q-p)}\,, \tag{53}$$

we obtain the final result for the normalized value of $\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N$ on the finite lattice

$$\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N = \mathrm{e}^{-2iht}\left[g(m,t)+\frac{\mathrm{d}}{\mathrm{d}z}\right]\det{}_N\left(\mathsf{S}_{ab} - z\mathsf{R}^{(+)}_{ab}\right)\Big|_{z=0}, \tag{54}$$

with the elements of the matrix S given by

$$\mathsf{S}_{ab} = \delta_{ab}d(p_a)e_-^2(p_a) + (1-\delta_{ab})\sin^2(\pi\kappa/2)\frac{e_+(p_a)e_-(p_b) - e_-(p_a)e_+(p_b)}{L\tan\frac{1}{2}(p_a-p_b)} - \frac{\sin^2(\pi\kappa/2)}{L}g(m,t)e_-(p_a)e_-(p_b). \tag{55}$$

5.3. Normalized mean value of $\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N$

The derivation of a determinant representation for $\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N$ is very similar to the one presented in the previous section. The main difference is that compared with equation (39) the resolution of identity contains now eigenstates with $N-1$ particles. Therefore

$$\begin{aligned}\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N &= \sum_{q_1<\cdots<q_{N-1}} \frac{\mathsf{F}_{N-1}(m_2,t_2,\{p\},\{q\})\overline{\mathsf{F}}_{N-1}(m_1,t_1,\{p\},\{q\})}{L^{2N-1}},\\ &= \frac{(\sin(\pi\kappa/2))^{2N-2}}{L^{2N-1}}\sum_{q_1<\cdots<q_{N-1}} \mathrm{e}^{\mathrm{i}t\left(\sum_{a=1}^N\varepsilon(p_a)-\sum_{b=1}^{N-1}\varepsilon(q_b)\right)-\mathrm{i}m\left(\sum_{a=1}^N p_a-\sum_{b=1}^{N-1}q_b\right)}\\ &\quad\times(\det{}_N\mathsf{B})^2\,. \end{aligned} \tag{56}$$

In the last line we have used that $\mathsf{F}_{N-1}(m,t,\{p\},\{q\})\equiv\langle\Psi_N(\{p\}|a^\dagger_m(t)|\Psi_{N-1}(\{q\})\rangle$ (note the interchange of $\{p\}$ and $\{q\}$) is given by

$\mathsf{F}_{N-1}(m,t,\{p\},\{q\}) = (\mathrm{i}\sin(\pi\kappa/2))^{N-1}\mathrm{e}^{\mathrm{i}t\left(\sum_{a=1}^N\varepsilon(p_a)-\sum_{b=1}^{N-1}\varepsilon(q_b)\right)-\mathrm{i}m\left(\sum_{a=1}^N p_a-\sum_{b=1}^{N-1}q_b\right)}\det{}_N\mathsf{B}$,

with $\mathrm{card}\{p\}=N$, $\mathrm{card}\{q\}=N-1$ and the $N\times N$ matrix B has the elements

$$\mathsf{B}_{ab}=\begin{cases}\cot\frac{1}{2}(p_a-q_b) & a=1,\cdots,N,\quad b=1,\cdots,N-1\,,\\ 1 & a=1,\cdots,N,\quad b=N\,.\end{cases} \tag{57}$$

Similar to the case treated in the previous section $(\det_N\mathsf{B})^2$ is a symmetric function of q's and vanishes when two of them coincide. Replacing the sum appearing in (56) with

$$\sum_{q_1<\cdots<q_{N-1}} = \frac{1}{(N-1)!}\sum_{q_1}\cdots\sum_{q_{N-1}},$$

and using $\det_N\mathsf{B}=\sum_{Q\in S_N}(-1)^{[Q]}\prod_{a=1}^{N-1}\cot\frac{1}{2}(p_{Q(a)}-q_a)$ we find

$$\begin{aligned}&\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N\\ &\quad= \frac{(\sin(\pi\kappa/2))^{2(N-1)}}{(N-1)!\,L^{2N-1}}\sum_{q_1,\cdots,q_{N-1}} f(\{p\},\{q\})\sum_{P,Q\in S_N}(-1)^{[P]+[Q]}\\ &\quad\times\prod_{a=1}^{N-1}\cot\frac{1}{2}(p_{P(a)}-q_a)\cot\frac{1}{2}(p_{Q(a)}-q_a)\\ &\quad\times\frac{(\sin(\pi\kappa/2))^{2(N-1)}}{(N-1)!\,L^{2N-1}}\sum_{q_1,\cdots,q_{N-1}} f(\{p\},\{q\})\sum_{Q,R\in S_N}(-1)^{[R]+[Q]+[Q]}\\ &\quad\times\prod_{a=1}^{N-1}\cot\frac{1}{2}(p_{RQ(a)}-q_a)\cot\frac{1}{2}(p_{Q(a)}-q_a)\\ &\quad\times\frac{(\sin(\pi\kappa/2))^{2(N-1)}}{(N-1)!\,L^{2N-1}}\sum_{q_1,\cdots,q_{N-1}} f(\{p\},\{q\})\\ &\quad\times\sum_{Q\in S_N}\begin{vmatrix}\cot\frac{1}{2}(p_{Q(1)}-q_1) & \cdots & \cot\frac{1}{2}(p_{Q(1)}-q_{N-1}) & 1\\ \vdots & \vdots & \vdots & \vdots\\ \cot\frac{1}{2}(p_{Q(N)}-q_1) & \cdots & \cot\frac{1}{2}(p_{Q(N)}-q_{N-1}) & 1\end{vmatrix}\\ &\quad\times\prod_{a=1}^{N-1}\cot\frac{1}{2}(p_{Q(a)}-q_a)\,, \end{aligned} \tag{58}$$

where $f(\{p\},\{q\}) = \mathrm{e}^{2ith}\prod_{a=1}^{N}(e_-(m,t,p_a))^2\prod_{b=1}^{N-1}(e_-(m,t,q_b))^{-2}$. Multiplying the i-th row ($i=1,\cdots,N$) of the determinant appearing in (58) with $e_-(p_{Q(i)})$, the j-th column ($j=1,\cdots,N-1$) with $\sin^2(\pi\kappa/2)\cot\frac{1}{2}(p_{Q(j)}-q_j)e_-(p_{Q(j)})/(L^2e_-^2(q_j))$ and the N-th column with $e_-(p_{Q(N)})/L$ and summing over q's inside the determinant (this is allowed because q_i appears only in the i-th column) we obtain

$$\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N = \frac{\mathrm{e}^{2ith}}{(N-1)!}$$
$$\times\sum_{Q\in S_N}\begin{vmatrix} \mathsf{U}_{Q(1)Q(1)}e_-(p_{Q(1)})e_-(p_{Q(1)}) & \cdots & \mathsf{U}_{Q(1)Q(N-1)}e_-(p_{Q(1)})e_-(p_{Q(N-1)}) & \mathsf{R}^{(-)}_{Q(1)Q(N)}\\ \vdots & \vdots & \vdots & \vdots\\ \mathsf{U}_{Q(N)Q(1)}e_-(p_{Q(N)})e_-(p_{Q(1)}) & \cdots & \mathsf{U}_{Q(N)Q(N-1)}e_-(p_{Q(N)})e_-(p_{Q(N-1)}) & \mathsf{R}^{(-)}_{Q(N)Q(N)}\end{vmatrix},$$

where we have introduced

$$\mathsf{R}^{(-)}_{ab} = \frac{e_-(p_a)e_-(p_b)}{L}. \tag{59}$$

Permuting the rows and columns with Q^{-1}, an operation which leaves the determinant unchanged, the sum over permutations gives $(N-1)!$ identical terms. The final result is

$$\langle a^\dagger_{m_2}(t_2)a_{m_1}(t_1)\rangle_N = \mathrm{e}^{2ith}\frac{\mathrm{d}}{\mathrm{d}z}\,\mathrm{det}_N(\mathsf{S}+z\mathsf{R}^{(-)})\Big|_{z=0} = \mathrm{e}^{2ith}[\mathrm{det}_N(\mathsf{S}+\mathsf{R}^{(-)})-\mathrm{det}_N\mathsf{S}]. \tag{60}$$

with S defined by (55).

5.4. Thermodynamic limit of the correlators

The final step in deriving the results presented in section 3 involves taking the thermodynamic limit in the finite lattice results obtained in the previous sections. Before we do that we remind the reader that the Fredholm determinant (see chapter XI of [61]) of an integral operator $\hat{K}$ which acts on an arbitrary function as $(\hat{K}f)(p)=\int_a^b K(p,q)f(q)\mathrm{d}q$ is an entire function of γ defined as

$$\begin{aligned}\det(1-\gamma\hat{K}) &= \lim_{n\to\infty}\left\{1-\gamma\sum_{j_1=1}^{n}\delta K(p_{j_1},p_{j_1})+\frac{\gamma^2}{2!}\sum_{j_1,j_2=1}^{n}\delta^2\begin{vmatrix}K(p_{j_1},p_{j_1}) & K(p_{j_1},p_{j_2})\\ K(p_{j_2},p_{j_1}) & K(p_{j_2},p_{j_2})\end{vmatrix}+\cdots\right\},\\ &= 1-\gamma\int_a^b K(p_1,p_1)dp_1+\frac{\gamma^2}{2!}\int_a^b\int_a^b\begin{vmatrix}K(p_1,p_1) & K(p_1,p_2)\\ K(p_2,p_1) & K(p_2,p_2))\end{vmatrix}\mathrm{d}p_1\mathrm{d}p_2+\cdots.\end{aligned} \tag{61}$$

Let us start with $\mathsf{g}_+^{(T)}(m,t)$. The thermodynamic limit of this correlator is

$$\mathsf{g}_+^{(T)}(m,t) = \lim_{L\to\infty}\frac{\sum_{N=0}^{L}\sum_{p_1<\cdots<p_N}\mathrm{e}^{-\frac{E_N(\{p\})}{T}}\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N}{\mathsf{tr}\ [\mathrm{e}^{-H/T}]}. \tag{62}$$

with $\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N$ given by (54). The denominator is

$$\begin{aligned}\mathsf{tr}\ [\mathrm{e}^{-H/T}] &= \sum_{N=0}^{L}\sum_{p_1<\cdots<p_N}\mathrm{e}^{-\frac{E_N(\{p\})}{T}} = \prod_p\left(1+\mathrm{e}^{-\varepsilon(p)/T}\right),\\ &\simeq \exp\left\{\frac{L}{2\pi}\int_{-\pi}^{+\pi}\ln(1+\mathrm{e}^{-\varepsilon(p)/T})\mathrm{d}p\right\}.\end{aligned} \tag{63}$$

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with $\varepsilon(p) = -2J\cos p + 2h$ and $L \to \infty$. This expression is divergent. A Fredholm determinant is well defined if the trace of the operator is finite $\int_{-\infty}^{+\infty} K(p,p)\mathrm{d}p < \infty$. Fortunately, as we will see below this divergent expression can be extracted from the numerator, making the expression for the correlator well defined. We can write that

$$\mathsf{tr}\,[\mathrm{e}^{-H/T}] = \det(1+\hat{\mathsf{Z}}), \quad \mathsf{Z}(p,p') = \mathrm{e}^{-\varepsilon(p)/T}\delta_L(p-p'),$$

where $\delta_L(p-p') = \frac{\sin(L(p-p'))}{2\pi(p-p')}$ is a regularization of the delta function. The numerator can be written as

$$\begin{aligned}
&\lim_{L\to\infty}\sum_{N=0}^{L}\sum_{p_1<\cdots<p_N} \mathrm{e}^{-\frac{E_N(\{p\})}{T}}\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N \\
&\quad= \lim_{L\to\infty}\sum_{N=0}^{L}\frac{1}{N!}\sum_{p_1}\cdots\sum_{p_N}\mathrm{e}^{-\frac{\sum_{a=1}^N\varepsilon(p_a))}{T}}\langle a_{m_2}(t_2)a^\dagger_{m_1}(t_1)\rangle_N, \\
&\quad= \lim_{L\to\infty}\sum_{N=0}^{L}\frac{1}{N!}\sum_{p_1}\cdots\sum_{p_N}\mathrm{e}^{-2iht}\left[g(m,t)+\frac{\partial}{\partial z}\right]\det{}_N(\tilde{\mathsf{S}}-z\tilde{\mathsf{R}}^{(+)})\Big|_{z=0}, \\
&\quad= \lim_{L\to\infty}\sum_{N=0}^{L}\frac{1}{N!}\sum_{p_1}\cdots\sum_{p_N}\mathrm{e}^{-2iht}\left[(g(m,t)-1)\det{}_N\tilde{\mathsf{S}}+\det{}_N(\tilde{\mathsf{S}}-\tilde{\mathsf{R}}^{(+)})\right],
\end{aligned} \tag{64}$$

with $\tilde{\mathsf{S}}$ and $\tilde{\mathsf{R}}^{(+)}$ obtained from (55) and (52) via the transformations $\tilde{\mathsf{S}}_{ab} = \mathrm{e}^{-\varepsilon(p_a)/2T}\,\mathsf{S}_{ab}\,\mathrm{e}^{-\varepsilon(p_b)/2T}$, $\tilde{\mathsf{R}}^{(+)}_{ab} = \mathrm{e}^{-\varepsilon(p_a)/2T}\,\mathsf{R}^{(+)}_{ab}\,\mathrm{e}^{-\varepsilon(p_b)/2T}$. Performing the thermodynamic limit and remembering that in this limit $e(p)\to E(p)$, $g\to G$, $d(p)\to D(p)$ (see (C3), (C5), (C7)) we obtain

$$\begin{aligned}
\mathsf{g}_+^{(T)}(m,t) &= \frac{\mathrm{e}^{-2iht}}{\det(1+\hat{\mathsf{Z}})}\left[G(m,t)+\frac{\partial}{\partial z}\right]\det(1+\hat{\mathsf{Z}}+\hat{\tilde{\mathsf{V}}}-z\hat{\tilde{\mathsf{R}}}^{(+)})\Big|_{z=0}, \\
&= \frac{\mathrm{e}^{-2iht}}{\det(1+\hat{\mathsf{Z}})}\left[(G(m,t)-1)\det(1+\hat{\mathsf{Z}}+\hat{\tilde{\mathsf{V}}})+\det(1+\hat{\mathsf{Z}}+\hat{\tilde{\mathsf{V}}}-\hat{\tilde{\mathsf{R}}}^{(+)})\right],
\end{aligned} \tag{65}$$

with kernels

$$\begin{aligned}
\tilde{V}(p,p') &= \sin^2\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{-\varepsilon(p)/2T}\mathrm{e}^{-\varepsilon(p')/2T}\left[\frac{E_+(p)E_-(p')-E_-(p)E_+(p')}{\tan[(p-p')/2]}\right. \\
&\qquad \left. -G(m,t)E_-(p)E_-(p')\right], \\
\tilde{R}^{(+)}(p,p') &= \sin^2\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{-\varepsilon(p)/2T}\mathrm{e}^{-\varepsilon(p')/2T}E_+(p)E_+(p').
\end{aligned} \tag{66}$$

It should be noted that the second term on the r.h.s of (C7) is obtained from the first term appearing in the square bracket of $\tilde{V}(p,p')$ in the limit $p\to p'$. Extracting $\det(1+\hat{\mathsf{Z}})$ from the numerator of (65) which cancels the similar term in the denominator we obtain the Fredholm determinant representation of $\mathsf{g}_+^{(T)}(m,t)$ (12). The thermodynamic limit of $\mathsf{g}_-^{(T)}(m,t)$ is performed along similar lines with the result (20).

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6. Conclusions

We have derived Fredholm determinant representations for the time-, space-, and temperature-dependent Green's function of a system of 1D hard-core anyons which can be understood as the fractional statistics generalization of hard-core bosons (XX0 spin chain). In the static case we have also computed the large distance asymptotics and the momentum distribution function at zero and finite temperature. The results obtained in this paper can be used to rigorously investigate the time-dependent correlation functions as in the case of the XX0 spin chain [53]. The first step would be the derivation of the classical integrable system of differential equations characterizing the correlators. Based on similar results for Lieb–Liniger anyons [12–14] we expect to obtain the same Ablowitz-Ladik system which characterizes the XX0 spin chain correlators [53] but with different boundary conditions. We should also point out that even though it is not as straightforward as in the static case, dynamic correlators can also be numerically investigated using the methods of [44]. This is left for further research.

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Appendix A: Finite size corrections

In this appendix we derive the finite size corrections of the low-lying excitations. In order to make the discussion below as simple and transparent as possible we consider the number of particles in the ground state to be N with N odd such that $\mathrm{e}^{\mathrm{i}\pi(1-\kappa)(N-1)}=1$. With this condition the BAEs for systems with N and $N+1$ particles are $\mathrm{e}^{\mathrm{i}p_aL}=\mathrm{e}^{-\mathrm{i}\pi\kappa(N-1)}=(-1)^{N-1}$, $a=1,\cdots,N$ and $\mathrm{e}^{\mathrm{i}q_aL}=\mathrm{e}^{-\mathrm{i}\pi\kappa N}=(-1)^N\mathrm{e}^{\mathrm{i}\pi(1-\kappa)}$, $a=1,\cdots,N+1$. The momenta of the particles in the ground states are

$$p_j=\frac{2\pi}{L}\left[-\frac{N+1}{2}+j\right],\quad j=1,\cdots,N, \tag{A1a}$$

$$q_j=\frac{2\pi}{L}\left[-\frac{N+2}{2}+\frac{1-\kappa}{2}+j\right],\quad j=1,\cdots,N+1\,. \tag{A1b}$$

We make a simple observation which will play an important role in the following discussion. We notice that BAEs of the system with N particles are the same as the BAEs of the XX0 spin chain [45] with the same number of quasiparticles. Therefore, if the momenta of the quasiparticles in the ground states of the XXO spin chain with N and $N+1$ quasiparticles are denoted by p_j^0 and q_j^0 we have $q_j=q_j^0$, $p_j=p_j^0+\frac{2\pi}{L}\frac{1-\kappa}{2}$.

In the thermodynamic limit at zero temperature we have $p_{j+1}-p_j=q_{j+1}-q_j=\frac{2\pi}{L}$ and the momenta fill densely the interval $[-k_{\mathrm{F}},k_{\mathrm{F}}]$ which means that we can replace sums with integrals using the rule $\sum\rightarrow\frac{L}{2\pi}\int_{-k_{\mathrm{F}}}^{k_{\mathrm{F}}}$. Due to the fact that the ground state with N particles is identical with the XX0 spin chain case we have $E(N,L)-Le_0=-\frac{\pi v_{\mathrm{F}}}{6L}c+O(1/L^2)$ with $c=1$ the central charge and $v_{\mathrm{F}}=\varepsilon'(k_{\mathrm{F}})=2J\sin k_{\mathrm{F}}$ the Fermi velocity. Now we are ready to compute the finite size corrections.

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A.1. Addition of one particle into the system

The momenta characterizing the ground states of systems with N and $N+1$ particles are given by (6). The momentum of this excitation is easily obtained $\Delta P(\Delta N=1) = 2k_{\rm F}(\frac{1-\kappa}{2}) + \frac{2\pi}{L}(\frac{1-\kappa}{2})$ due to $\sum p_j^0 = \sum q_j^0 = 0$ and $k_{\rm F} = \pi D$ with $D = N/L$. Introducing the notation $\omega = \frac{2\pi}{L}\frac{1-\kappa}{2}$ the energy of the excitation is

$$\begin{aligned}
\Delta E(\Delta N=1) &= \sum_{j=1}^{N+1} \varepsilon(q_j) - \sum_{j=1}^{N} \varepsilon(p_j)\,,\\
&= \sum_{j=1}^{N+1} \left[\varepsilon(q_j^0) + \varepsilon'(q_j^0)\omega + \varepsilon''(q_j^0)\frac{\omega^2}{2}\right] - \sum_{j=1}^{N}\varepsilon(p_j^0)\,,\\
&= \Delta E^0(\Delta N=1) + \frac{L}{2\pi}\int_{-k_{\rm F}}^{k_{\rm F}} \varepsilon'(q)\omega + \varepsilon''(q)\frac{\omega^2}{2}\,{\rm d}q\,,\\
&= \frac{2\pi v_{\rm F}}{L}\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1-\kappa}{2}\right)^2\right]
\end{aligned}$$

where we have used the finite size correction formula for the XXO spin chain (chap. II of [50] with $\mathcal{Z}=1$), $\Delta E^0(\Delta N=1) = \frac{\pi v_{\rm F}}{2L}$ and the fact that $\varepsilon'(q)$ is an odd function which gives zero after the integration over a symmetric interval.

A.2. Particle-hole excitation

We consider first a particle-hole excitation at the right Fermi boundary $k_{\rm F}$. The momenta are still given by (6) with the exception of $\tilde{q}_{N+1}$ which is $\tilde{q}_{N+1} = q_{N+1} + \frac{2\pi}{L}N^+$ with N^+ an arbitrary integer. The momentum of the excitation is simply $\Delta P(N^+) = \frac{2\pi N^+}{L}$ and the energy $\Delta E(N^+) = \varepsilon(\tilde{q}_{N+1}) - \varepsilon(q_{N+1}) = \frac{2\pi v_{\rm F}}{L}N^+$ using $\varepsilon(\tilde{q}_{N+1}) = \varepsilon(q_{N+1}) + \varepsilon'(q_{N+1})\frac{2\pi}{L}N^+$. Similar formulas, with N^+ replaced by N^-, can be derived in the case of a particle-hole excitation at the left Fermi boundary $-k_{\rm F}$.

A.3. Backscattering of *d* particles

In this case d particles jump from $-k_{\rm F}$ to $k_{\rm F}$. The momenta of the particles in the excited state are $\tilde{q}_j = \frac{2}{\pi L}\left[-\frac{N+2}{2} + \frac{1-\kappa}{2} + d + j\right] = q_j^0 + \omega + \omega'$ with $\omega' = \frac{2\pi}{L}d$. The momentum of the excitation is $\Delta P(d) = 2k_{\rm F}d + \frac{2\pi}{L}d$ and the energy

$$\begin{aligned}
\Delta E(d) &= \sum_{j=1}^{N+1}\varepsilon(\tilde{q}_j) - \sum_{j=1}^{N+1}\varepsilon(q_j),\\
&= \sum_{j=1}^{N+1}\left[\varepsilon(q_j^0) + \varepsilon'(q_j^0)(\omega+\omega') + \varepsilon''(q_j^0)\frac{(\omega+\omega')^2}{2}\right] - \sum_{j=1}^{N+1}\left[\varepsilon(q_j^0) + \varepsilon'(q_j^0)\omega + \varepsilon''(q_j^0)\frac{\omega^2}{2}\right],\\
&= \frac{2\pi v_{\rm F}}{L}\left[\left(d + \frac{1-\kappa}{2}\right)^2 - \left(\frac{1-\kappa}{2}\right)^2\right].
\end{aligned}$$

Collecting all the results we obtain equation (29).

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Appendix B: Wavefunctions expression for the form factor

In this appendix we derive equation (42) which is the starting point in our calculation of the form factors. It is instructive to consider first the simple case of $N = 2$ which contains all the relevant features associated with anyonic statistics. The generalization to any value of N is straightforward. Starting with the eigenvectors for two and three particles

$$|\Psi_2(\{p\})\rangle = \frac{1}{\sqrt{2!}}\sum_{m_1=1}^{L}\sum_{m_2=1}^{L}\chi_2(m_1,m_2|(\{p\})a_{m_2}^\dagger a_{m_1}^\dagger|0\rangle\,,$$

$$\langle\Psi_3(\{q\})| = \frac{1}{\sqrt{3!}}\sum_{n_1=1}^{L}\sum_{n_2=1}^{L}\sum_{n_3=1}^{L}\overline{\chi}_3(n_1,n_2,n_3|\{q\})\langle 0|a_{n_1}a_{n_2}a_{n_3}\,,$$

we obtain for the static form factor the following expression

$$\begin{aligned}\mathsf{F}_2(m,\{q\},\{p\}) = \frac{1}{\sqrt{2!3!}}&\sum_{n_1,n_2,n_3=1}^{L}\sum_{m_1,m_2=1}^{L}\overline{\chi}_3(n_1,n_2,n_3|\{q\})\chi_2(m_1,m_2|(\{p\})\\ &\times\langle 0|a_{n_1}a_{n_2}a_{n_3}a_m^\dagger a_{m_2}^\dagger a_{m_1}^\dagger|0\rangle\,.\end{aligned} \tag{B1}$$

Moving successively the annihilation operators to the right with the help of the commutation relations (2) and using $a_j|0\rangle = 0$ we find

$$\begin{aligned}\langle 0|a_{n_1}a_{n_2}a_{n_3}\quad a_m^\dagger a_{m_2}^\dagger a_{m_1}^\dagger|0\rangle &= \delta_{n_3,m}\delta_{n_2,m_2}\delta_{n_1,m_1} - \mathrm{e}^{-\mathrm{i}\pi\kappa\epsilon(n_2-m_2)}\delta_{n_3,m}\delta_{n_1,m_2}\delta_{n_2,m_1}\\ &- \mathrm{e}^{-\mathrm{i}\pi\kappa\epsilon(n_3-m)}\delta_{n_3,m_2}\delta_{n_2,m}\delta_{n_1,m_1} + \mathrm{e}^{-\mathrm{i}\pi\kappa[\epsilon(n_3-m)+\epsilon(n_2-m)]}\delta_{n_3,m_2}\delta_{n_2,m_1}\delta_{n_1,m}\\ &+ \mathrm{e}^{-\mathrm{i}\pi\kappa[\epsilon(n_3-m)+\epsilon(n_3-m_2)]}\delta_{n_3,m_1}\delta_{n_1,m_2}\delta_{n_2,m}\\ &- \mathrm{e}^{-\mathrm{i}\pi\kappa[\epsilon(n_3-m)+\epsilon(n_3-m_2)+\epsilon(n_2-m)]}\delta_{n_3,m_1}\delta_{n_2,m_2}\delta_{n_1,m}\,.\end{aligned}$$

Plugging this relation in (B1) and using the anyonic symmetry of the wavefunctions (5) we obtain

$$\mathsf{F}_2(m,\{q\},\{p\}) = \sqrt{3}\sum_{m_1=1}^{L}\sum_{m_2=1}^{L}\sum_{m_3=1}^{L}\overline{\chi}_3(m_1,m_2,m|\{q\})\chi_2(m_1,m_2|(\{p\}).$$

The generalization of this result in the case of N particles is given by equation (42).

Appendix C: Thermodynamic limit of singular function

Here we calculate the thermodynamic limit of the functions introduced in section 5. On the finite lattice the allowed values for q's are given by $q_l = \frac{2\pi}{L}\left(-\frac{L}{2}+l\right) + \frac{2\pi\delta'}{L}$ with $l = 1,\cdots,L$. In the thermodynamic limit $q_{l+1} - q_l = 2\pi/L$ and the q's fill densely the interval $[-\pi,\pi]$, which means that the sums appearing in the definition of the various functions defined in section 5 can be replaced by integrals using the following rule

$$\frac{1}{L}\sum_q \to \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}q\,. \tag{C1}$$

In the case of functions involving differences of the type $q - p$ we need to distinguish between the $\langle a_{m_2}(t_2)a_{m_1}^\dagger(t_1)\rangle_N$ and $\langle a_{m_2}^\dagger(t_2)a_{m_1}(t_1)\rangle_N$ cases, which will be denoted by

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$\langle aa^\dagger\rangle$ and $\langle a^\dagger a\rangle$. This is because the BAEs satisfied by the q's are $\mathrm{e}^{\mathrm{i}qL} = \mathrm{e}^{-\mathrm{i}\pi\kappa N}$ ($\langle aa^\dagger\rangle$ case) and $\mathrm{e}^{\mathrm{i}qL} = \mathrm{e}^{-\mathrm{i}\pi\kappa(N-2)}$ ($\langle a^\dagger a\rangle$ case) with $\mathrm{e}^{\mathrm{i}pL} = \mathrm{e}^{-\mathrm{i}\pi\kappa(N-1)}$ in both cases. Therefore,

$$\sum_q f(q-p) = \begin{cases} \sum_{j=1}^{L} f[\frac{2\pi}{L}(j-k-\kappa/2)]\,, & k\in\{1,\cdots,L\}\text{for}\langle aa^\dagger\rangle\,,\\ \sum_{j=1}^{L} f[\frac{2\pi}{L}(j-k+\kappa/2)]\,, & k\in\{1,\cdots,L\}\text{for}\langle a^\dagger a\rangle\,. \end{cases} \tag{C2}$$

The simplest situation is encountered in the case of the $g(m,t)$ function. Using (C1) we have

$$g(m,t) = \frac{1}{L}\sum_q \mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q} \longrightarrow G(m,t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathrm{d}q\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q}\,. \tag{C3}$$

The first nontrivial case is encountered in the case of $e(m,t,p) = \frac{1}{L}\sum_q \frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q}}{\tan\frac{1}{2}(q-p)}$. On the finite lattice this function is well defined because $q\neq p$. However, in the thermodynamic limit $q\sim p$ and the function will have a singularity. We want to extract the singular part. Using formula 4.4.7 (1) on page 646 of [62], $\sum_{k=1}^{n-1}\cot(x+k\pi/n) = n\cot(nx)$ we find

$$\sum_q \frac{1}{\tan\frac{1}{2}(q-p)} = \sum_{j=1}^{L}\frac{1}{\tan\frac{\pi}{L}(j-k\pm\kappa/2)} = \pm L\cot\left(\frac{\pi\kappa}{2}\right)\,, \tag{C4}$$

where the plus (minus) sign corresponds to $\langle aa^\dagger\rangle$ ($\langle a^\dagger a\rangle$). Separating the singular part in the form

$$e(m,t,p) = \frac{1}{L}\sum_q \frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q} - \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\tan(\frac{q-p}{2})} + \frac{1}{L}\sum_q\frac{\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\tan(\frac{q-p}{2})}\,,$$

the thermodynamic limit of $e(m,t,p)\to E(m,t,p)$ is

$$\begin{aligned} E(m,t,p) &= \frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathrm{d}q\,\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q} - \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\tan\frac{1}{2}(q-p)} \pm \cot\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}\,,\\ &= \mathrm{PV}\frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathrm{d}q\,\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q}}{\tan\frac{1}{2}(q-p)} \pm\cot\left(\frac{\pi\kappa}{2}\right)\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p} \end{aligned} \tag{C5}$$

where in the last line we have used the identity $\mathrm{PV}\frac{1}{2\pi}\int_{-\pi}^{+\pi}\mathrm{d}q/\tan\frac{1}{2}(q-p) = 0$ and the plus (minus) sign corresponding to $\langle aa^\dagger\rangle$ ($\langle a^\dagger a\rangle$).

The $d(m,t,p,\kappa)$ function can be written as

$$d(m,t,p,\kappa) = \frac{\sin^2(\pi\kappa/2)}{L^2}\left[\sum_q\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q} - \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\sin^2\frac{1}{2}(q-p)} + \sum_q\frac{\mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\sin^2\frac{1}{2}(q-p)}\right]\,.$$

The sum appearing as the last term in the square parenthesis can be computed using the formula 4.4.6 (9) on page 645 of [62], $\sum_{k=0}^{n-1}\sin^{-2}(x+k\pi/n) = n^2\sin^{-2}(nx)$, with the result

$$\sum_q\frac{1}{\sin^2\frac{1}{2}(q-p)} = \sum_{j=1}^{L}\frac{1}{\sin^2(\frac{\pi}{L}(j-k\pm\kappa/2))} = \frac{L^2}{\sin^2\left(\frac{\pi\kappa}{2}\right)}\,, \tag{C6}$$

Introducing $f(p) = \sum_q\frac{\mathrm{e}^{\mathrm{i}mq+2\mathrm{i}Jt\cos q} - \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}}{\tan\frac{1}{2}(q-p)}$ and $l(p) = \mathrm{e}^{\mathrm{i}mp+2\mathrm{i}Jt\cos p}$ it can be shown that

$$d(m,t,p,\kappa) = \frac{2\sin^2(\pi\kappa/2)}{L^2}\left[\frac{\mathrm{d}f(k)}{\mathrm{d}k} + \frac{\mathrm{d}l(k)}{\mathrm{d}k}\sum_q\frac{1}{\tan\frac{1}{2}(q-p)}\right] + l(k)\,,$$

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which together with (C4) and (C5) shows that in the thermodynamic limit $d(m,t,p,\kappa) \to D(m,t,p,\kappa)$ is given by

$$D(m,t,p,\kappa) = \mathrm{e}^{imp+2\mathrm{i}Jt\cos p} + \frac{2\sin^2(\pi\kappa/2)}{L}\frac{\partial}{\partial k}E(m,t,p)\,. \tag{C7}$$

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