# Large Scale Structure without Dark Matter

## *TeVeS, relativistic gravitation theory for the MOND paradigm*

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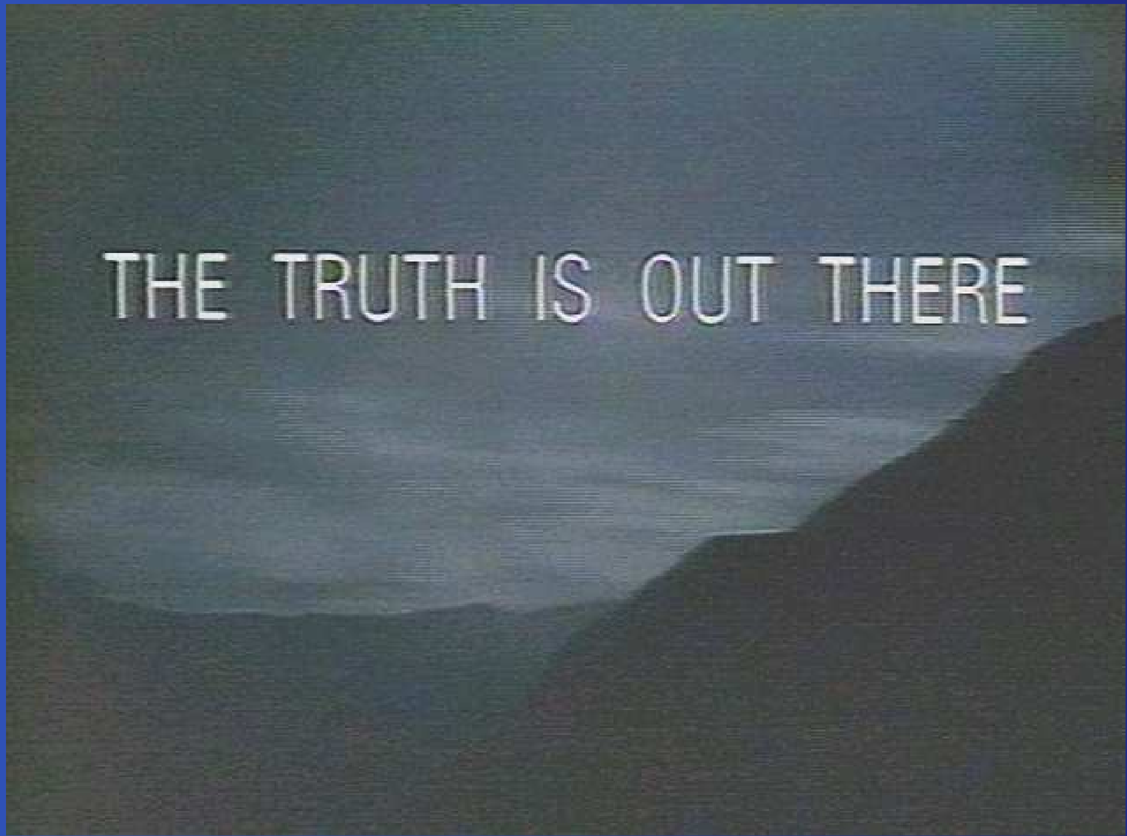
# Alternative Gravities vs. Dark Matter



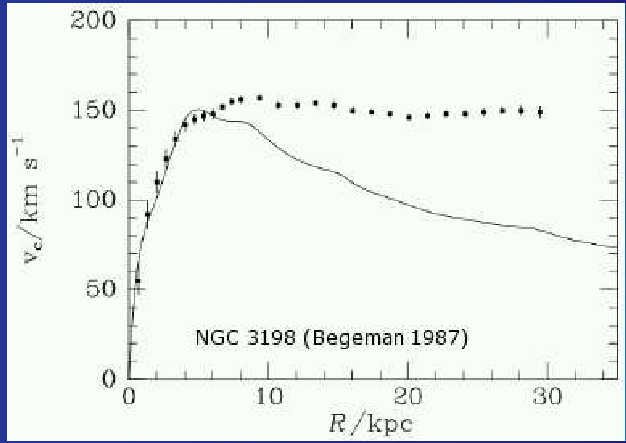

Each galaxy has its own history of formation-evolution-interaction!

# Outline

- Milgrom's MOND paradigm
- Bekenstein's proposal for a covariant theory: TeVeS
- Cosmology with TeVeS
- Some remarks...

# Milgrom's MOND paradigm

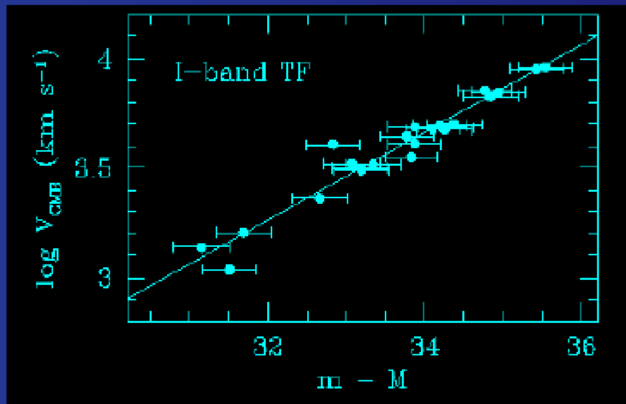
The "missing mass" problem or "acceleration discrepancy": Galaxy Rotation Curve - Expected and Observed




Two observational facts about spiral galaxies:

- rotation curves are asymptotically flat
- Tully-Fisher law (1977): rotation velocity in spirals vs. luminosity

$$L \propto V^4$$



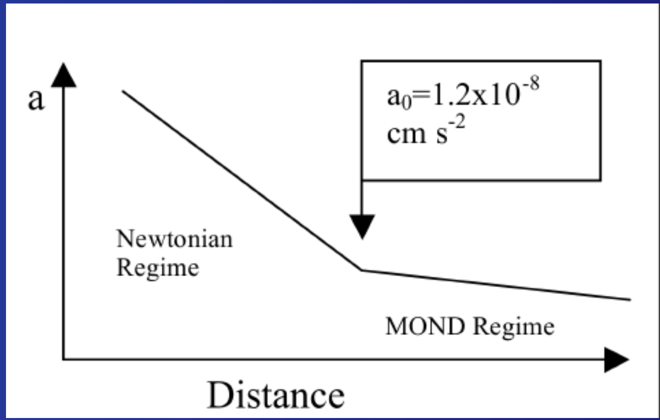
# Milgrom's MOND paradigm

- Discrepancies between the Newtonian dynamical mass and the direct observabile mass in large systems has two possible explanations:
  - either these systems contain DM
  - or gravity on these scales is not described by Newtonian theory
- MOdified Newtonian Dynamics (MOND) has been proposed by Milgrom (1983) as an alternative to the DM:
  - as a particle's acceleration approaches some limiting small value $a_0$, Newton's second law breaks down

$$m\,\mu\left(\frac{a}{a_0}\right)\mathbf{a} = \mathbf{F}$$

# Milgrom's MOND paradigm

$$\begin{cases} a \gg a_o & \Rightarrow a_N = \dfrac{GM}{r^2} \\ a \ll a_o & \Rightarrow a = \dfrac{\sqrt{GMa_0}}{r} \end{cases}$$



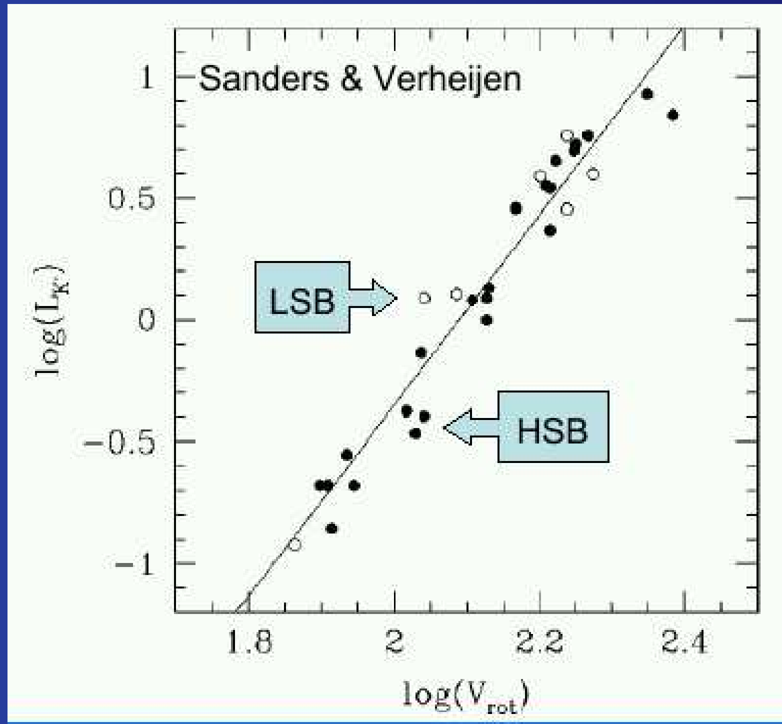

An interpolation function derived empirically joints the two regimes:
$\mu(x \ll 1) \approx x$ and $\mu(x \gg 1) \to 1$ with $x = \frac{a}{a_0}$

If we set the effective gravitational force $g = \sqrt{g_N a_0}$ equal to the centripetal acceleration $V^2/r$, then

$V^4 = GMa_0!$ TF mass-velocity relation

# Milgrom's MOND paradigm

- The observed TF relation for Ursa Major spirals (Sanders & Verheijen 1998) gives $a_0 \approx 10^{-8}$ cm s$^{-2}$.
- Note: data for LSB galaxies became available some 10 years later after Milgrom made its predictions.



# Milgrom's MOND paradigm

**FIRST** high-resolution simulation of the structure formation in a MONDian universe: Knebe & Gibson, 2004

Adhere to an existing Poisson solver mapping $a_N$ to $a$ by using the cosmological $N$-body code MLAPM (open source); AMIGA

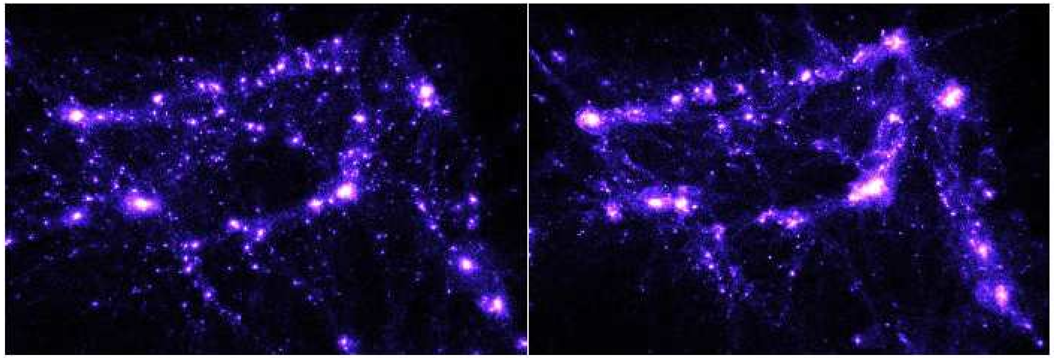


Figure 1: ΛCDM cosmological simulation at redshift $z = 0$ (left panel) vs. a simulation incorporating MOND and using the same initial conditions (right panel).

- for $z > 4 - 5$ their simulations show less galaxies than data
- influence of the cosmological constant $\Omega_\lambda$ is ignored
- do not include the last results of Bekenstein (2004) and Skordis et al. (2006)

# Milgrom's MOND paradigm

- MOND can be interpretated as providing an universal profile of dark halos $a_0\, f[GM/a_0 r^2] = GM_{DM}(r)/r^2$, but CDM does not involve this universal scale $a_0$.
- $a_0$ is of order of $cH_0 \Rightarrow$ Cosmological origin of $a_0$??
- Many successes (e.g., rotation curves & TF relation) but problems (cluster mass to light too large).
- MOND also predicts: LBS galaxies are DM dominated, no DM in HBS; no DM in the center of the galaxies (cusp problem in CDM).
- MOND is not a theory; it violates conservation of energy and of angular momentum.
- MOND is not complete: it does not specify haw to calculate gravitational lensing by galaxies and cluster of galaxies.

# TeVeS theory

How to build relativistic theory – basic idea

$$\mu\vec{a} = \vec{\nabla}\Phi_N \qquad \nabla^2\Phi_N = 4\pi G\rho$$

$$\vec{a} = -\vec{\nabla}\Phi \qquad \vec{\nabla}\cdot\left[f\left(\frac{\nabla\Phi}{a_0}\right)\vec{\nabla}\Phi\right] = 4\pi G\rho$$

Dynamics — Gravity

Relativistic version:

Dynamics (geodesic eqs.) — Gravity (Einstein eqs.)

$$a^\mu + \Gamma^\mu_{\alpha\beta}v^\alpha v^\beta = 0 \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nwarrow \quad \nearrow$$

$$g_{\mu\nu}$$

common metric

# TeVeS theory

“Bimetric" theories – use two different metrics:

$g_{\mu\nu}$ — “Physical" metric in geodesic eqs

$\tilde{g}_{\mu\nu}$ — “Geometric" metric in Einstein eqs.

Bekenstein, 2004:

$$g_{\mu\nu} = e^{-2\phi}\left(\tilde{g}_{\mu\nu} + A_\mu A_\nu\right) - e^{2\phi} A_\mu A_\nu$$

- 2 Tensor fields ($g_{\mu\nu}$, $\tilde{g}_{\mu\nu}$) + 1 Vector field ($A_\mu$) + 1 Scalar field ($\phi$) there is also a nondynamical scalar field $\mu$
- $A_\mu$ is a time-like 4-vector field s.t. $\tilde{g}^{\mu\nu} A_\mu A_\nu = 1$; $A_\mu = (-\sqrt{g_{00}}, 0, 0, 0)$

# TeVeS theory

Total action of the system: $S = S_g + S_s + S_v + S_m$

$$
\begin{aligned}
S_g &= \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\tilde{R} \\
S_s &= -\frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left[\mu\left(\tilde{g}^{\mu\nu} - A^\mu A^\nu\right)\phi_{,\alpha}\phi_{,\beta} + V(\mu)\right] \\
S_v &= -\frac{1}{32\pi G}\int d^4x\sqrt{-\tilde{g}}\left[K_B F^{\alpha\beta}F_{\alpha\beta} - 2\lambda(A^\mu A_\mu + 1)\right] \\
S_m &= \int d^4x\sqrt{-g}L_m\left[g_{\mu\nu}, \chi^A, \nabla\chi^A\right]
\end{aligned}
$$

$K_B$ = dimensionless constant, $\lambda$ = Lagrange multiplier, $F_{\alpha\beta} = 2\nabla_{[\alpha}A_{\beta]}$,
$\mu$ = nondynamical field with a free function $V(\mu)$

IMPORTANT!! TeVeS theory lies within the choice of the function $V(\mu)$!!

# TeVeS theory

- A choice of $V$ will pick out a given theory!!
- Bekenstein proposal:

$$V = \frac{3\mu_0^2}{128\pi l_B^2}\left[\hat{\mu}(4 + 2\hat{\mu} - 4\hat{\mu}^2 + \hat{\mu}^3) + 2ln(\hat{\mu} - 1)^2\right]$$

  This potential will lead to the prescription proposed by Milgrom in the non-relativistic regime.

- There are 3 free parameters that appear in the TeVeS total action: $\mu_0$, $l_B$,, and $K_B$.
- Armed with this total action, Bekenstein (2004) and Skordis et al. (2006) solved for the evolution of the scale factor $a$ of a homogeneous Friedmann-Robertson-Walker metric.

# Cosmology with TeVeS

Skordis et al., 2006

- Homogeneous and isotropic spacetimes
- Two metrics: two scale factors...$a = be^{-\phi}$

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  Physical Geometric
- Choose a coordinate system s.t. $A^{\alpha} = (1, 0, 0, 0)$
- Modified Friedmann equation becomes

$$\tilde{H}^2 = \frac{8\pi G_{eff}}{3}(\rho + \rho_\phi)$$

where $G_{eff} = G\frac{e^{-4\phi}}{(1+\frac{d\phi}{d\ln a})^2}$ and $\rho_\phi = \frac{e^{2\phi}}{16\pi G}(\mu V' + V)$

# Cosmology with TeVeS

- TeVeS modifications to the standard cosmology depend on the evolution of the scalar field $\phi$.
- Ordinary fluid energy density $\rho_X$ evolves as usual as

$$\dot{\rho}_X = -3\frac{\dot{a}}{a}(1+w)\rho_X \ ,$$

  where $w$ is the equation of state parameter of the fluid.
- Relative densities $\Omega_i$ are defined as usual as $\Omega_i = \dfrac{\rho_i}{\rho_i + \rho_\phi}$.
- Variation of the action with respect to the field $\mu$ gives $\dot{\phi}^2 = \frac{1}{2}V'$.
- Finally, the scalar field evolves according to

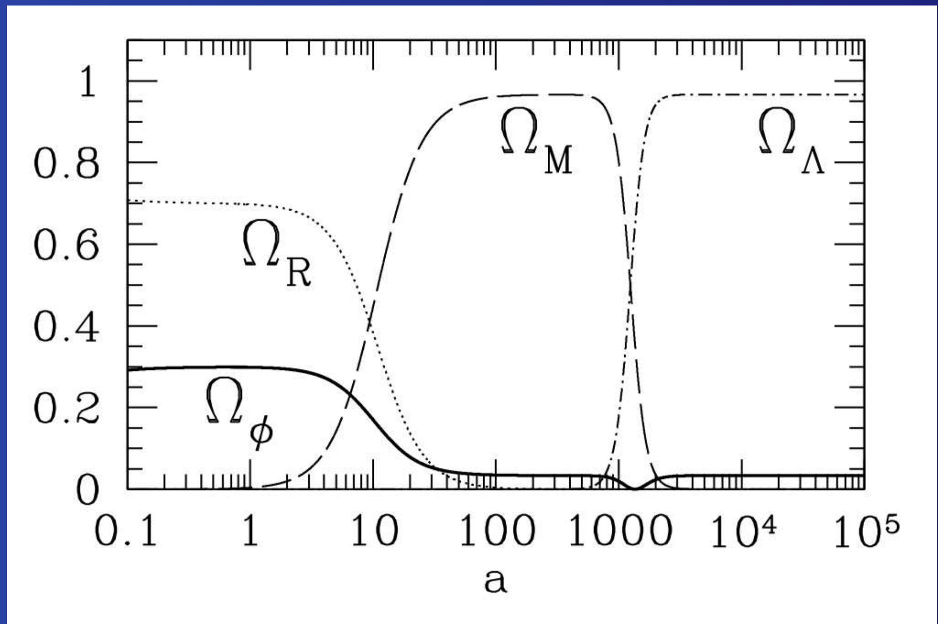
$$\dot{\phi} = -\frac{1}{2\tilde{\mu}}\Gamma, \text{ where } \dot{\Gamma} + 3\tilde{H}\Gamma = 8\pi G e^{-\phi}\rho_X(1+3w)$$

# Cosmology with TeVeS

**Cosmological tracking:**

If $V' \propto \frac{\mu^2(\mu - 2\mu_0)}{\mu - \mu_0} \Rightarrow \Omega_\phi = \begin{cases} \frac{3}{2\mu_0} & \text{radiation epoch} \\ \frac{1}{6\mu_0} & \text{matter epoch} \end{cases}$

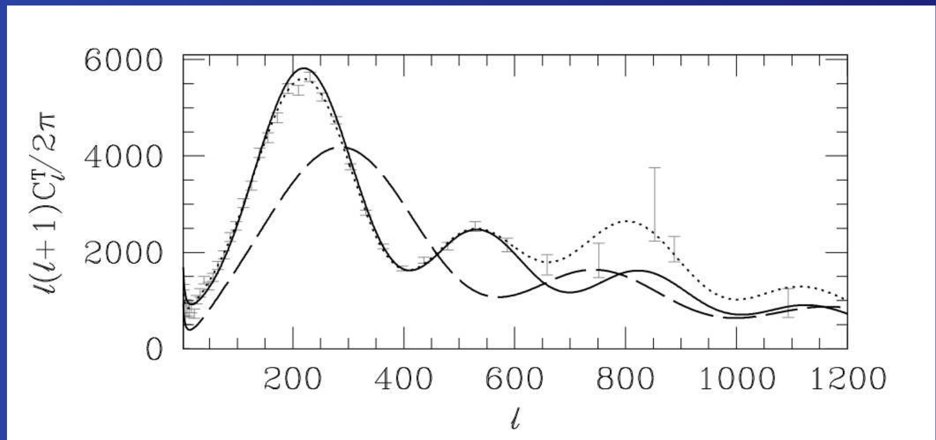
**BBN:** $\Omega_\phi < 10^{-4}$
**Not Dark Matter!**



# Cosmology with TeVeS

- **Perturbations** to the metric, matter, radiation, and TeVeS fields are governed by a set of coupled differential equations
- perturbations in the scalar field may induce enhanced growth in the matter perturbation – oscillate in the radiation epoch (Skordis et al., 2006)
- MOND universes compared to the CMB data

1. MOND universe with $a_0 = 4.2 \times 10^{-8}$ cm/s$^2$: $\Omega_V = 0.17$, $\Omega_\nu = 0.17$, and $\Omega_\nu = 0.05$ (solid line) $\Omega_V = 0.95$ and $\Omega_\nu = 0.05$ (dashed line)
2. $\Lambda$CDM model (dotted line)



# Some remarks...

- The success of MOND's phenomenology may signal a break down of the Newtonian gravity at small accelerations.
- Any competing model of DM should explains the success of the MOND, and the existence of an universal acceleration scale.
- Does the Bullet Cluster rule out the MOND paradigm?
- TeVeS-like models suffer from instabilities of the vector field; moreover, the free functions must to be fine-tuned.
- These difficulties may signal that MOND needs a more general framework than the (pseudo-)Riemannian geometry.
- Looking forward for any surprise!