Dusty Plasma in Space Sciences

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The dust is omnipresent in nature. In many situations is a disturbing factor, as in our daily life. But sometimes it is responsible for new interesting phenomena taking place in space. The DUSTY PLASMA can be described as a complex system of electrons, different species of ions and negatively or positively charged particles of dust. The presence of charged dust particles can generate new and unusual behaviors in a plasma (new collective modes of oscillations, instabilities, new coherent nonlinear structures). For strongly interacting dusty plasma, the dust grains can organize into a dusty crystal configuration. The state of art of the field was discussed in many review articles and books and we mention only two of them where many other references can be found [1],[2].

The dusty plasma is rather ubiquitous in space, and its history is quite old. We shall present below, very briefly, some situations where dusty plasma plays a dominant role.

COMETS

A bright comet is an excellent cosmic laboratory for the study of dust-plasma interactions, and their physical and dynamical consequences. As the comet approaches the Sun, at few AU, the nucleus is warming up and a long tail of evaporated molecules, carrying small solid particles, is formed. The Sun’s radiation pressure and the solar wind accelerate in different ways the components of comet’s tail. The ion tail appears as a more or less straight line, opposite to the Sun, while the dust tail is slower accelerated, being much broader and tends to be curved, as one easily sees in the picture of the Hale-Bopp comet below.
PLANETARY RINGS

It is well established that most of the rings of the giant planets (Jupiter, Saturn, Neptun) are made of micron to submicron sized dust particles.

The rings of Saturn, discovered by Galileo in 1610, are still a non completely understood problem nowadays. The spatial fly-bys of Voyager 1 and 2 (1980-81) and Cassini-Huygens mission (orbiting Saturn since 2004) have revealed many unexpected things. The main rings are named A, B and C (see figure 2).
The particles in Saturn’s rings contain primarily ice and are from microns to meters in size. One of the most interesting features observed was the existence of nearly radial spokes, especially in the ring B (extending from $1.5 \, R_S$ to $1.95 \, R_S - R_S$, the radius of Saturn). They have an inner limit at about $1.72 \, R_S$ and extend up to the outer edge of the B ring. A typical spoke pattern is presented in figure 3. It is assumed that the spokes are containing electrostatically levitated micron and submicron sized dust grains, and the thin radial elongation is due to the rapid radial motion of dense plasma clouds.
Earth’s Atmosphere

The presence of dust particles in Earth’s atmosphere was observed during the polar summer mesopause, at altitudes between 80 and 90 km. The most significant phenomenon is the formation of a special type of clouds, named *noctilucent clouds* (from latin - night shining). They were first observed in 1885 after the Krakatoa vulcan eruption. The NLC are composed of ice, heavy ion clusters, dust from man made pollution (terrestrial aerosols), rockets and space shuttle exhausts (mainly $Al_2O_3$ spheroids of 0.1 to 10 $\mu$m in size). They are visible only when illuminated by sunlight from below the horizon, while the ground and lower layers of the atmosphere are in the Earth’s shadow, as one can see in figure 4.

A beautiful picture is presented in the followings

![Image of noctilucent clouds](image)
Dusty Plasma in Laboratories

The dusty plasma is present also in laboratory d.c. and r.f. discharges. It originates from the plasma chemistry in gas phase or from sputtering of the electrodes. Also a large quantity of macroscopic dust grains are found in plasma processing reactors. It can be easily diagnosed using laser light scattering. The dust particles have a cauliflower like structure on the outside and a columnar interior microstructure.

The presence of dust particles in fusion devices was known for a long time. The plasma in tokamacs, stelaratons are more or less contaminated; the impurities are generated by several processes such as desorption, arching, sputtering, evaporation and sublimation of thermally overloaded wall material. They are influencing the operation and performances of the devices.

Characteristics of a Dusty Plasma

Characteristic lengths:

- \( r_d \) – dust grain radius
- \( a \) – average intergrain distance
- \( \lambda_D \) – plasma Debye radius

If \( r_d \ll \lambda_D < a \) the dust particles can be considered as a collection of isolated grains (each charged dust grain is strongly screened and they are only weakly interacting). We can speak about dust in plasma.

If \( r_d \ll a < \lambda_D \) the charged dust particles are strongly interacting (they are not enough screened) and are participating to the collective behavior of the plasma. This is real dusty plasma.

The presence of dust grains modifies the existing low-frequency waves and can introduce also some new types of excitations (linear and nonlinear)

- dust acoustic waves (DAW)
- dust ion-acoustic wave (DIAW)
- dust acoustic (DA) solitons/shocks
- dust ion-acoustic (DIA) solitons/shocks

Macroscopic neutrality writes

\[
q_i n_i 0 = e n_e 0 - q_d n_d 0
\]

\( n_e 0, n_i 0, n_d 0 \) – the equilibrium densities of electrons, ions and dust particles;
\( q_i = Z_i e \) – ion charge (usually \( Z_i = 1 \);
\( q_d = -Z_d e \) - dust charge (negatively charged). Here \( Z_d \) is the number of elementary charges on the dust grain surface (usually of the order \( 10^5 \)) and although \( n_{d0} \ll n_0 \) we can have \( Z_d n_{d0} \simeq n_{i0} \).

**Debye shielding**

One of the main abilities of a plasma is to shield the electric field of an individual charged particle. A negatively charged dust particle attracts around it the positive ions and a cloud of ions is formed. The edge of this cloud is situated at a radius where the electrostatically potential energy is approximately equal with the thermal energy \( k_B T_d \) of the dust particles. As \( m_d/m_i \gg 1 \), the dust particles can be considered at rest. Also we can assume that \( n_e \) and \( n_i \) are in equilibrium with the local potential \( \Phi \)

\[
n_e = e_0 \exp \left( \frac{e\Phi}{k_B T_e} \right), \quad n_i = n_{i0} \exp \left( -\frac{e\Phi}{k_B T_i} \right)
\]

(2)

The Poisson equation writes

\[
\nabla^2 \Phi = 4\pi (e n_e - e n_i - q_d n_d)
\]

(3)

Here \( q_d \) is an average value of the dust charge, assumed independent of time. Also we assume that \( \frac{e\Phi}{k_B T_{e,i}} \ll 1 \) in the most part of the cloud. Then after a series expansion of the exponential in (2) and using the neutrality condition (1), we get

\[
\Phi = \Phi_0 \exp \left( -\frac{r}{\lambda_D} \right)
\]

\[
\lambda_D = \frac{\lambda_D e \lambda_{Di}}{\sqrt{\lambda_D^2 + \lambda_{Di}^2}}
\]

(4)

\[
\lambda_D^2 = \frac{k_B T_{e,i}}{4\pi e^2 n_{e,i} 0}
\]

In most laboratory experiments \( n_{i0} \ll n_0 \) (the electrons stick on the dust particles and consequently the electron cloud is strongly depopulated) and \( T_e \simeq T_i \) so \( \lambda_{De} \gg \lambda_{Di} \) and the screening is mostly determined by the ions, \( \lambda_D \simeq \lambda_{Di} \).

**Characteristic frequency**

A small perturbation in the equilibrium properties gives rise to collective motions (oscillations) which tend to restore the original charge neutrality. We have to solve together the equations of continuity for each species (\( s = e, i, d \))

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{V}_s) = 0,
\]

(5)
the equations of motion
\[
\frac{\partial \vec{V}_s}{\partial t} + (\vec{V}_s \cdot \nabla) \vec{V}_s = -\frac{q_s}{m_s} \nabla \Phi,
\]
and Poisson’s equation
\[
\nabla^2 \Phi = -4\pi \sum_s q_s n_s
\]
Here \(n_s\) is the density of species \(s\), \(\vec{V}_s\) the velocity of the corresponding fluid, and \(\Phi\) the potential. Writing \(n_s = n_{s0} + n_{s1}\), where \(n_{s1} \ll n_{s0}\) we get
\[
\frac{d^2 \Phi}{dt^2} + \omega_p^2 \Phi = 0
\]
Here \(\omega_p\) is the plasma frequency associated to each species \(s\). The physical picture is the following: the electrons oscillate around the ions with the frequency \(\omega_{pe}\), the ions oscillate around the charged dust particles with the frequency \(\omega_{pi}\), and the dust particles around their equilibrium positions with the frequency \(\omega_{pd}\).

**Coulomb coupling parameter**

Considering two dust grains, of the same charge \(q_d\), separated by a distance \(a\), the Coulomb potential energy (including the shielding effect) is given by
\[
E_c = \frac{q_d^2}{a} \exp \left( -\frac{a}{\lambda_D} \right)
\]
The Coulomb parameter is defined as
\[
\Gamma_c = \frac{E_c}{k_B T_d} = \frac{Z_d^2 e^2}{a k_B T_d} \exp \left( -\frac{a}{\lambda_D} \right)
\]
If \(\Gamma_c \ll 1\) we have a weakly coupled dusty plasma, while for \(\Gamma_c \gg 1\) the dusty plasma is strongly coupled. In the case of a strongly coupled dusty plasma, new phenomena, like the formation of Coulomb crystals, may occur (when \(\Gamma_c \geq 170\)) [3].

**Dust charging process**

The central point in the physics of a dusty plasma is to understand the charging process of dust grains immersed in an ambient plasma and with a radiative background. The main processes are:
- The interaction of dust grains with gaseous plasma particles. The interaction with electrons (mostly) leads to negatively charged grains, but in certain conditions the interaction can be with the positive ions leading to positively charged grains.
- The interaction between dust grains and energetic particles, resulting in a secondary electron emission and leading to positively charged grains.
- The interaction with photons which by photoemission charge the grains positively.
- Other processes, like thermionic emission, field emission, and impact ionization with neutrals usually lead to positively charged dust grains.

The different charging processes are carefully discussed in [2].

Linear waves in a dusty plasma (DA Waves)

A great variety of collective wave phenomena arises due to coherent motion of plasma constituents. In what follows we shall consider only unmagnetized plasma. A typical longitudinal wave which can exist in a plasma is the ion-acoustic wave arising from the fluctuations of the electrical potential. The presence of charged dust particles can modify, and in certain range of frequency even dominate the wave propagation. As mentioned before, the dust acoustic (DA) and dust ion-acoustic (DIA) waves are specific for a dusty plasma.

DA waves were first discussed by Rao, Shukla and Yu in 1990 [4],[2]. This kind of waves are expected to appear in the low frequency limit and a good approximation is to consider the electrons and the positive ions in equilibrium with the local potential. Denoting by $n_{e1}$ and $n_{i1}$ the deviations of the electron/ion densities from the equilibrium values we have

$$n_{e1} = n_{e0} \frac{e \Phi}{k_B T_e}, \quad n_{i1} = -n_{i0} \frac{e \Phi}{k_B T_i}$$ (12)

where $\Phi$ is the local potential. The fluid of dust particles, characterized by the density $n_d$ and velocity $\vec{V}_d$ satisfies a continuity and a momentum equation

$$\frac{\partial n_d}{\partial t} + n_d \nabla \cdot \vec{V}_d = 0$$ (13)

$$\frac{\partial \vec{V}_d}{\partial t} = -\frac{q_d}{m_d} \nabla \Phi - \frac{3k_B T_d}{m_d n_{d0}} \nabla n_{d1}$$
to which we have to add the Poisson equation

$$\nabla^2 \Phi = 4\pi (en_{1} - q_{d0}n_{d1} - en_{1})$$  \hspace{1cm} (14)$$

The last term in the right hand side of the second eq. (13) is a pressure term. We assumed the same charge for all dust particles (constant in time). The system (13), (14) is easily solved assuming plane wave solutions

$$n_{d1}, \Phi_{1} \sim \exp \left[ i \left( k \vec{r} - \omega t \right) \right]$$

The following dispersion relation for DA waves results

$$1 + \frac{k_{D}^{2}}{k^{2}} - \frac{\omega_{pd}^{2}}{\omega^{2} - 3k^{2}V_{Td}^{2}} = 0$$  \hspace{1cm} (15)$$

where $k_{D} = 1/\lambda_{D}$ while $V_{Td}$ is the thermal velocity of the dust particles. Since $\omega \gg kV_{Td}$, from (15) one obtains

$$\omega = \frac{kC_{D}}{\sqrt{1 + k^{2}\lambda_{D}^{2}}}, \quad C_{D} = \lambda_{D}\omega_{pd}$$

which in the long wave length limit shows that the wave has an acoustic character ($\omega \sim k$). The observed DA wave frequencies are of order 10-20 Hz and video images are possible and can be seen with naked eye.

These results can be improved taking into account the effects of boundaries and collisions with neutral atoms, which can modify the dispersion properties of DA waves [2].

**Nonlinear Structures in Dusty Plasma**

Usually the small amplitude excitations of the dusty plasma, found in the linear theory, are unstable to small modulations of the amplitude. Therefore the amplitude is growing and soon the nonlinear mechanisms have to be taken into account. But there are numerous nonlinear processes via which unstable modes can saturate, and stable, robust excitations are formed. Here we shall consider a class of such mechanisms, involving the competition between dispersion and nonlinearity. For a common cold plasma, its behavior is described by the well known Korteweg-de Vries equation (KdV), which is a completely integrable system having soliton solutions. When the dissipative effect is comparable, or dominant in respect to the dispersive one, one encounters shock waves described by a KdV-Burgers type equation. The presence of dust grains introduces new features, and new nonlinear structures, like dust acoustic solitary waves and solitons can exist. In the following, we shall discuss the properties of the dust acoustic solitons (DAS) waves.  

9
We consider a dusty plasma containing hot electrons \((T_e)\) and ions \((T_i)\) in thermodynamic equilibrium \((\sigma = T_i/T_e)\) with densities (normalized by the equilibrium values \((n_{e0}, n_{i0}, \delta = n_{i0}/n_{e0} > 1)\).

\[
n_i = \exp(-\varphi), \quad n_e = \exp(\sigma \varphi)
\]  

(16)

Here \(\varphi\) is the local potential measured in units \(k_B T_i/e\). In a 1-D geometry and adimensional variables \(z, t\) (space variable \(z\) measured in units of Debye length \(\lambda_D = (k_B T_i/4\pi e^2 Z_d n_0)^{1/2}\), time \(t\) variable measured in units \(\omega_p^{-1}\), \(\omega_p^2 = 4\pi n_0 (Z_d e)^2/m_d\), \(m_d\) the mass of the dust particle, \(n_0\) equilibrium density, \((-Z_d e)\) the negative constant charge of the dust particles), the space and time evolution for the dust density \(n\) and velocity \(u\) (measured in units \(C_d = (Z_d k_B T_i/m_d)^{1/2}\) is described by

\[
\begin{align*}
\frac{\partial n}{\partial t} + \frac{\partial}{\partial z} (nu) &= 0 \\
\frac{\partial \varphi}{\partial t} + u \frac{\partial u}{\partial z} &= \frac{\partial \varphi}{\partial z} \\
\frac{\partial^2 \varphi}{\partial z^2} &= n + \mu_e n_e - \mu_i n_i
\end{align*}
\]  

(17)

where

\[
\mu_e = \frac{n_{e0}}{Z_d n_0} = \frac{1}{\delta - 1}, \quad \mu_i = \frac{n_{i0}}{Z_d n_0} = \frac{\delta}{\delta - 1}
\]  

(18)

The charge neutrality in the equilibrium state is assumed to be satisfied

\[
n_{e0} + Z_d n_0 - n_{i0} = 0
\]  

(19)

The effect of nonlinearity is a cumulative one and manifests at long time and space scales. Appropriate asymptotic methods are necessary to treat this effect, namely the “reductive perturbation technique” used for the first time by Washimi and Taniuti many years ago [5], [4], [2] to derive the KdV equation in plasma physics.

One introduces the stretched variables

\[
\xi = \varepsilon^{1/2} (x - v_0 t), \quad \tau = \varepsilon^{3/2} t
\]  

(20)

and expand \(n, u, \varphi\) in powers of \(\varepsilon\)

\[
\begin{align*}
n &= 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \ldots \\
u &= \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots \\
\varphi &= \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \ldots
\end{align*}
\]  

(21)

Using these in eqs. (17) we get in order \(\varepsilon^{3/2}\)

\[
v_0 = \frac{1}{\sqrt{\mu_i + \mu_e \sigma}}
\]  

(22)
while in order $\varepsilon^{5/2}$, after eliminating the second order quantities $n^{(2)}$, $u^{(2)}$, $\varphi^{(2)}$ we remain with the following KdV equation which $\varphi^{(1)}$ has to satisfy

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + a\varphi^{(1)} \frac{\partial \varphi^{(1)}}{\partial \xi} + b\frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = 0. \quad (23)$$

Here

$$a = \frac{v_i^2}{2} \left( \mu_i - \sigma_i^2 \mu_e - \frac{3}{v_0^2} \right), \quad b = \frac{v_0^3}{2} \quad (24)$$

As it is well known, the KdV equation (23) has the traveling solution (1-soliton solution)

$$\varphi^{(1)} = \varphi_m^{(1)} \text{sech}^2 \left[ (\xi - u_0 \tau) / \Delta \right]$$

$$\varphi_m^{(1)} = \frac{3u_0}{a} \quad \Delta^2 = \frac{4b}{u_0} \quad (25)$$

Here $u_0$ is the wave velocity in $\xi$-space. If $a > 0$ we have $\varphi_m^{(1)} > 0$, but for $a < 0$, $\varphi_m^{(1)} < 0$. An explicit expression of $a$ is

$$a = -\frac{v_0^3}{(\delta - 1)^2} \left[ \delta^2 + (3\delta + \sigma)\sigma + \frac{1}{2}\delta(1 + \sigma^2) \right] < 0$$

so the potential in a DA soliton is negative.

Many extensions are possible [2]; the following effects have been considered:

- effect of dust fluid temperature;
- effect of the trapped ion distribution;
- effect of dust charge fluctuation;
- other geometries, cylindrical and spherical DAS;
- effect of the ponderomotive force of high-frequency waves on plasma slow motions.

To conclude we can say that the study of dusty plasma remains a very active field of research, with many applications in space sciences, but also of a growing interest in industrial applications (deposition, etching, ...), and in fusion devices. Many theoretical problems are still unsolved or less understood. It is a very complex system that still has many subjects left open for investigation.
References


