Heliospheric Electric and Magnetic Fields

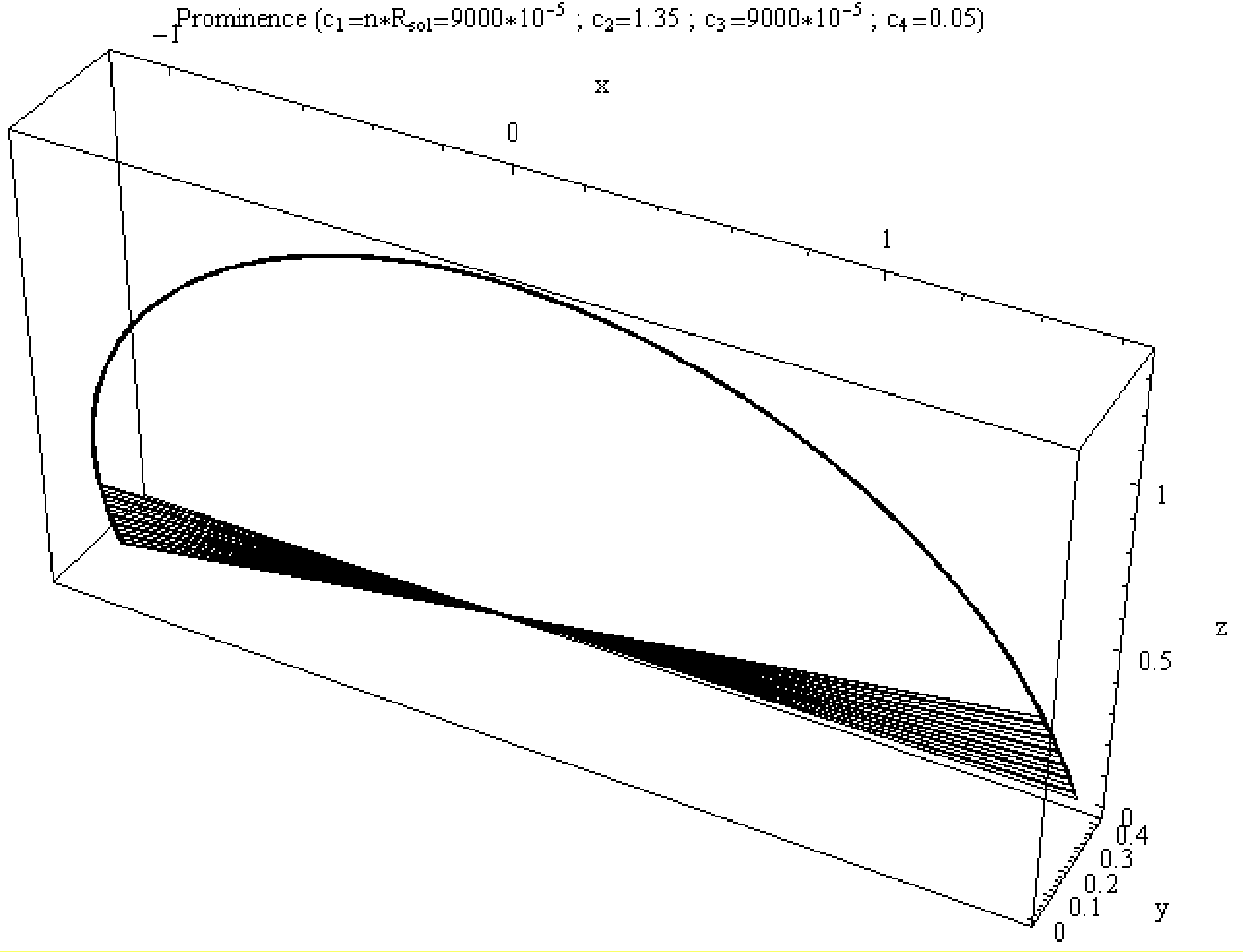
Adrian Sabin Popescu

Astronomical Institute of the Romanian Academy

Introduction

$$\begin{cases} r(t_{FRW}) = c_2 \exp\left\{-c_1 \sinh\left[\arctan\left(\dfrac{1}{\tan t_{FRW}}\right)\right]\right\} \\ \zeta(t_{FRW}) = -\arctan\left(\dfrac{1}{\tan t_{FRW}}\right) \\ \xi(t_{FRW}) = c_4 - c_3 \sinh\left[\arctan\left(\dfrac{1}{\tan t_{FRW}}\right)\right] . \end{cases}$$

Introduction



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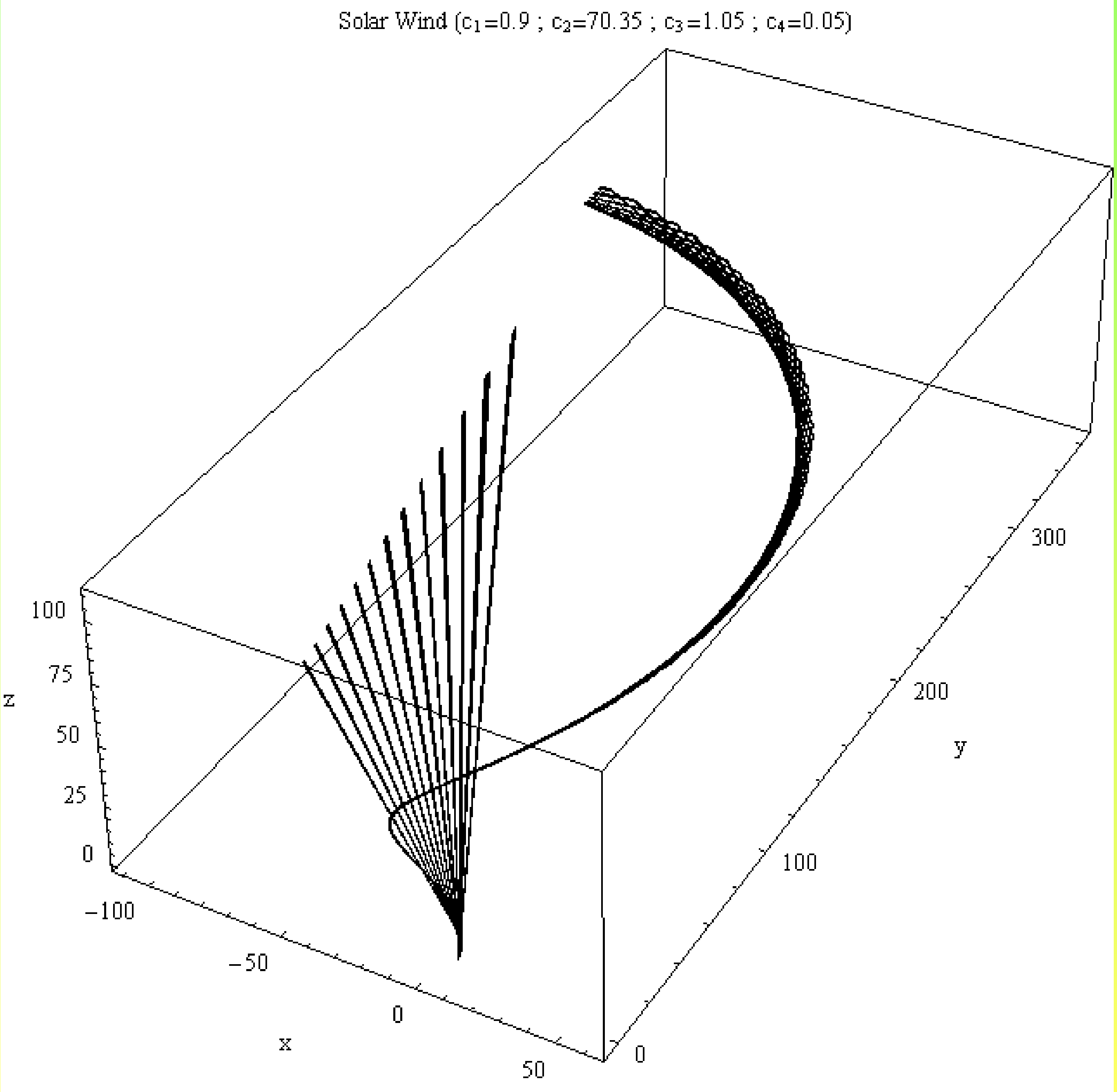

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Introduction

From the Maxwell equations in the local Minkowski spacetime chart (derived from the DEUS topology) we obtain the relations to be particularized for a solar type star and a massive star, and later to be used for a 3D representation of the electric and magnetic field topology (in heliosphere or in a stellar atmosphere) and of its evolution with the cosmological time.

The goal of this study is modest: to obtain the equation background of a code meant to follow the evolution of the electric and magnetic field topology in heliosphere (or solar type star atmosphere) and in the atmosphere of a massive star (or black hole). The difference between the two cases will come from the choice of the local Minkowski coordinate chart.

MAXWELL EQUATIONS

In Minkowski spacetime, in curvilinear coordinates:

$$\nabla \times \mathbf{E} = \mp \frac{2}{\sqrt{3}} \frac{1}{r}\left[\frac{1}{\tan\zeta} E_\xi + \frac{\partial E_\xi}{\partial \zeta} - \frac{1}{\sin\zeta}\frac{\partial E_\zeta}{\partial \xi}\right]\vec{e_r} \mp \mp \frac{2}{\sqrt{3}} \frac{1}{r}\left[\frac{1}{\sin\zeta}\frac{\partial E_r}{\partial \xi} - E_\xi - r\frac{\partial E_\xi}{\partial r}\right]\vec{e_\zeta} \mp \frac{2}{\sqrt{3}}\frac{1}{r}\left[E_\zeta + r\frac{\partial E_\zeta}{\partial r} - \frac{\partial E_r}{\partial \zeta}\right]\vec{e_\xi}\,. \tag{1}$$

We have also:

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\partial t_{FRW}}{\partial t}\left[\frac{\partial B_r}{\partial t_{FRW}}\vec{e_r} + \frac{\partial B_\zeta}{\partial t_{FRW}}\vec{e_\zeta} + \frac{\partial B_\xi}{\partial t_{FRW}}\vec{e_\xi}\right] = = \pm\frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\left[\frac{\partial B_r}{\partial t_{FRW}}\vec{e_r} + \frac{\partial B_\zeta}{\partial t_{FRW}}\vec{e_\zeta} + \frac{\partial B_\xi}{\partial t_{FRW}}\vec{e_\xi}\right]. \tag{2}$$

By combining (1) and (2) in the Faraday's law of induction results:

$$\frac{2}{\sqrt{3}}\frac{1}{r}\left[\frac{1}{\tan\zeta}E_\xi + \frac{\partial E_\xi}{\partial \zeta} - \frac{1}{\sin\zeta}\frac{\partial E_\zeta}{\partial \xi}\right] = \frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\frac{\partial B_r}{\partial t_{FRW}}\,, \tag{3}$$

$$\frac{2}{\sqrt{3}}\,\frac{1}{r}\left[\frac{1}{\sin\zeta}\,\frac{\partial E_r}{\partial\xi}-E_\xi-r\,\frac{\partial E_\xi}{\partial r}\right]=\frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{\partial B_\zeta}{\partial t_{FRW}}\,, \tag{4}$$

$$\frac{2}{\sqrt{3}}\,\frac{1}{r}\left[E_\zeta+r\,\frac{\partial E_\zeta}{\partial r}-\frac{\partial E_r}{\partial\zeta}\right]=\frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{\partial B_\xi}{\partial t_{FRW}}\,. \tag{5}$$

In the same manner, we can write the Gauss law of magnetism:

$$\nabla.\mathbf{B}=\pm\frac{2}{\sqrt{3}}\,\frac{1}{r}\left[r\,\frac{\partial B_r}{\partial r}\,\vec{e_r}+\left(\frac{1}{\tan\zeta}\,B_\zeta+\frac{\partial B_\zeta}{\partial\zeta}\right)\vec{e_\zeta}+\frac{1}{\sin\zeta}\,\frac{\partial B_\xi}{\partial\xi}\,\vec{e_\xi}\right]=0\,, \tag{6}$$

resulting:

$$\frac{\partial B_r}{\partial r}=0\,, \tag{7}$$

$$\frac{1}{\tan\zeta}\,B_\zeta+\frac{\partial B_\zeta}{\partial\zeta}=0\,, \tag{8}$$

$$\frac{\partial B_\xi}{\partial\xi}=0\,. \tag{9}$$

This means that:

$$\frac{\partial B_r}{\partial t_{FRW}} = \frac{\partial B_r}{\partial r}\,\frac{\partial r}{\partial t_{FRW}} = 0\,, \tag{10}$$

which gives in (3):

$$\frac{1}{\tan\zeta}\,E_\xi + \frac{\partial E_\xi}{\partial \zeta} - \frac{1}{\sin\zeta}\,\frac{\partial E_\zeta}{\partial \xi} = 0\,, \tag{11}$$

and:

$$\frac{\partial B_\xi}{\partial t_{FRW}} = \frac{\partial B_\xi}{\partial \xi}\,\frac{\partial \xi}{\partial t_{FRW}} = 0\,, \tag{12}$$

which gives in (5):

$$E_\zeta + r\,\frac{\partial E_\zeta}{\partial r} - \frac{\partial E_r}{\partial \zeta} = 0\,. \tag{13}$$

Also, with:

$$\frac{\partial B_\zeta}{\partial t_{FRW}} = \frac{\partial B_\zeta}{\partial \zeta}\,\frac{\partial \zeta}{\partial t_{FRW}} = -\frac{1}{\tan\zeta}\,B_\zeta\,, \tag{14}$$

where $\frac{\partial \zeta}{\partial t_{FRW}} = 1$, in (4), we get:

$$\frac{2}{\sqrt{3}}\,\frac{1}{r}\left[\frac{1}{\sin\zeta}\,\frac{\partial E_r}{\partial \xi}-E_\xi-r\,\frac{\partial E_\xi}{\partial r}\right]=-\frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{1}{\tan\zeta}\,B_\zeta\,. \qquad (15)$$

When we consider the coordinates for a solar type star, as function of t_{FRW}, the relation (11) can be written as:

$$-\tan t_{FRW}\,E_\xi+\frac{\partial E_\xi}{\partial t_{FRW}}+\frac{1}{\sin\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{1}{c_3\,\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{\partial E_\zeta}{\partial t_{FRW}}=0\,, \qquad (16)$$

relation (13) can be written as:

$$E_\zeta+\frac{1}{c_1\,\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{\partial E_\zeta}{\partial t_{FRW}}-\frac{\partial E_r}{\partial t_{FRW}}=0\,, \qquad (17)$$

and relation (15) as:

$$-\frac{2}{\sqrt{3}}\,\frac{1}{c_2\,\exp\left\{-c_1\,\sinh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\right\}}\left\{\frac{1}{c_3\,\sin\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]}\,\frac{\partial E_r}{\partial t_{FRW}}+\right.$$

$$\left.+\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]E_\xi+\frac{1}{c_1}\,\frac{\partial E_\xi}{\partial t_{FRW}}\right\}=\tan t_{FRW}\,B_\zeta\ . \tag{18}$$

Now, in natural units, in the Ampère's law we have $\mu_0\epsilon_0 = 1/c^2 = 1$. The electric current density will be described by $\mathbf{J} = \rho\,\mathbf{v}$ or, with the help of Gauss law for electricity:

$$\mathbf{J} = \epsilon_0\,\mathbf{v}(\nabla.\mathbf{E})\ . \tag{19}$$

In (19), the velocity $\mathbf{v} = dx^i/dt$, with $x^i = \{r,\zeta,\xi\}$ and t the global DEUS time:

$$\frac{dt}{dt_{FRW}} = \pm\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\ . \tag{20}$$

Then, we can write the Ampère's law as:

$$\nabla\times\mathbf{B} = \mathbf{v}(\nabla.\mathbf{E}) + \frac{\partial\mathbf{E}}{\partial t}\ . \tag{21}$$

In our curvilinear Minkowski coordinates:

$$\nabla \times \mathbf{B} = \mp \frac{2}{\sqrt{3}} \frac{1}{r}\left[\frac{1}{\tan\zeta} B_\xi + \frac{\partial B_\xi}{\partial \zeta} - \frac{1}{\sin\zeta} \frac{\partial B_\zeta}{\partial \xi}\right]\vec{e_r} \mp$$

$$\mp \frac{2}{\sqrt{3}} \frac{1}{r}\left[\frac{1}{\sin\zeta} \frac{\partial B_r}{\partial \xi} - B_\xi - r \frac{\partial B_\xi}{\partial r}\right]\vec{e_\zeta} \mp \frac{2}{\sqrt{3}} \frac{1}{r}\left[B_\zeta + r \frac{\partial B_\zeta}{\partial r} - \frac{\partial B_r}{\partial \zeta}\right]\vec{e_\xi}\,, \qquad (22)$$

or, using the (7) and (9) results of the Gauss law for magnetism:

$$\nabla \times \mathbf{B} = \mp \frac{2}{\sqrt{3}} \frac{1}{r} \left[\frac{1}{\tan\zeta} B_\xi - \frac{1}{\sin\zeta} \frac{\partial B_\zeta}{\partial \xi} \right] \vec{e_r} \mp \frac{2}{\sqrt{3}} \frac{1}{r} \left[-B_\xi \right] \vec{e_\zeta} \mp \frac{2}{\sqrt{3}} \frac{1}{r} \left[B_\zeta + r \frac{\partial B_\zeta}{\partial r} \right] \vec{e_\xi} . \tag{23}$$

We can explicitely write also:

$$\mathbf{v}(\nabla.\mathbf{E}) = \pm \frac{2}{\sqrt{3}} \frac{\partial t_{FRW}}{\partial t} \left[\frac{\partial E_r}{\partial t_{FRW}} \vec{e_r} - \frac{\tan t_{FRW}}{r} E_\zeta \vec{e_\zeta} + \frac{1}{r} \frac{\partial E_\zeta}{\partial t_{FRW}} \vec{e_\zeta} + \frac{1}{r \sin\zeta} \frac{\partial E_\xi}{\partial t_{FRW}} \vec{e_\xi} \right] . \tag{24}$$

Then, from the (21) law, with (8), we obtain:

$$\frac{\partial E_r}{\partial t_{FRW}} = \mp \frac{2}{\sqrt{3}} \frac{\partial t}{\partial t_{FRW}} \frac{1}{1 \pm \frac{2}{\sqrt{3}}} \frac{1}{r} \left[\frac{1}{\tan\zeta} B_\xi + \frac{1}{\sin\zeta} \frac{1}{\tan\zeta} B_\zeta \frac{\partial t_{FRW}}{\partial \xi} \right] , \tag{25}$$

$$B_\xi = \frac{\partial t_{FRW}}{\partial t} \left[-\tan t_{FRW} E_\zeta + \frac{\partial E_\zeta}{\partial t_{FRW}} \right] \pm \frac{\sqrt{3}}{2} r \frac{\partial E_\zeta}{\partial t_{FRW}} \frac{\partial t_{FRW}}{\partial t} , \tag{26}$$

$$\frac{\partial E_\xi}{\partial t_{FRW}} = \mp \frac{2}{\sqrt{3}} \frac{\partial t}{\partial t_{FRW}} \frac{\sin\zeta}{r \sin\zeta \pm \frac{2}{\sqrt{3}}} \left[1 - \frac{r}{\tan\zeta} \frac{\partial t_{FRW}}{\partial r} \right] B_\zeta . \tag{27}$$

With (26) in (25) we get:

$$\frac{\partial E_r}{\partial t_{FRW}} = \mp\frac{2}{\sqrt{3}}\,\frac{1}{1\pm\dfrac{2}{\sqrt{3}}}\,\frac{1}{r}\left\{\frac{1}{\tan\zeta}\left[-\tan t_{FRW}\,E_\zeta + \frac{\partial E_\zeta}{\partial t_{FRW}}\right]\pm \right.$$
$$\left. \pm\frac{\sqrt{3}}{2}\,\frac{r}{\tan\zeta}\,\frac{\partial E_\zeta}{\partial t_{FRW}} + \frac{1}{\sin\zeta}\,\frac{1}{\tan\zeta}\,B_\zeta\,\frac{\partial t_{FRW}}{\partial \xi}\,\frac{\partial t}{\partial t_{FRW}}\right\}. \tag{28}$$

Expressing equation (13) with the help of (28), we have:

$$E_\zeta + r\,\frac{\partial E_\zeta}{\partial t_{FRW}}\,\frac{\partial t_{FRW}}{\partial r} \pm \frac{2}{\sqrt{3}}\,\frac{1}{1\pm\dfrac{2}{\sqrt{3}}}\,\frac{1}{r}\left\{\frac{1}{\tan\zeta}\left[-\tan t_{FRW}\,E_\zeta + \frac{\partial E_\zeta}{\partial t_{FRW}}\right]\pm \right.$$
$$\left. \pm\frac{\sqrt{3}}{2}\,\frac{r}{\tan\zeta}\,\frac{\partial E_\zeta}{\partial t_{FRW}} + \frac{1}{\sin\zeta}\,\frac{1}{\tan\zeta}\,B_\zeta\,\frac{\partial t_{FRW}}{\partial \xi}\,\frac{\partial t}{\partial t_{FRW}}\right\} = 0\,. \tag{29}$$

Equation (11) with (27) becomes:

$$\frac{1}{\tan\zeta}\,E_\xi \mp \frac{2}{\sqrt{3}}\,\frac{\partial t}{\partial t_{FRW}}\,\frac{\sin\zeta}{r\,\sin\zeta\pm\dfrac{2}{\sqrt{3}}}\left[1-\frac{r}{\tan\zeta}\,\frac{\partial t_{FRW}}{\partial r}\right]B_\zeta - \frac{1}{\sin\zeta}\,\frac{\partial E_\zeta}{\partial t_{FRW}}\,\frac{\partial t_{FRW}}{\partial \xi} = 0\,. \tag{30}$$

From equation (15) results:

$$B_\zeta = -\frac{2}{\sqrt{3}}\frac{1}{r}\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\tan\zeta\left[\frac{1}{\sin\zeta}\frac{\partial E_r}{\partial t_{FRW}}\frac{\partial t_{FRW}}{\partial \xi} - E_\xi - r\frac{\partial E_\xi}{\partial t_{FRW}}\frac{\partial t_{FRW}}{\partial r}\right], \tag{31}$$

and then, with (27) and (28):

$$\begin{aligned} B_\zeta = &-\frac{2}{\sqrt{3}}\frac{1}{r}\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\tan\zeta\left\{1 \mp \frac{2}{\sqrt{3}}\frac{1}{r}\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\tan\zeta \times\right. \\ &\times\left[\frac{1}{\sin^2\zeta}\frac{2}{\sqrt{3}}\frac{1}{1\pm\frac{2}{\sqrt{3}}}\frac{1}{r}\frac{1}{\tan\zeta}\left(\frac{\partial t_{FRW}}{\partial \xi}\right)^2\frac{\partial t}{\partial t_{FRW}} - r\frac{2}{\sqrt{3}}\frac{\partial t}{\partial t_{FRW}}\frac{\sin\zeta}{r\sin\zeta\pm\frac{2}{\sqrt{3}}}\times\right. \\ &\left.\left.\times\left(1-\frac{r}{\tan\zeta}\frac{\partial t_{FRW}}{\partial r}\right)\frac{\partial t_{FRW}}{\partial r}\right]\right\}^{-1}\left\{\mp\frac{1}{\sin\zeta}\frac{2}{\sqrt{3}}\frac{1}{1\pm\frac{2}{\sqrt{3}}}\frac{1}{r}\times\right. \\ &\left.\times\left[-\frac{\tan t_{FRW}}{\tan\zeta}E_\zeta + \frac{1}{\tan\zeta}\frac{\partial E_\zeta}{\partial t_{FRW}} \pm \frac{\sqrt{3}}{2}\frac{r}{\tan\zeta}\frac{\partial E_\zeta}{\partial t_{FRW}}\right]\frac{\partial t_{FRW}}{\partial \xi} - E_\xi\right\}. \end{aligned} \tag{32}$$

CONCLUSIONS

Using the (r,ζ,ξ) coordinates for the solar type star or for a massive star (or black hole) we can study how the electric and magnetic field components vary according to the t_{FRW} cosmic time.

With (32) in (29) and (30), we are able to determine E_ζ and E_ξ, from where, back in (32), B_ζ and, in (28), E_r. As resulting from the Gauss law for magnetism, B_ξ and B_r are constant through the t_{FRW} cosmologic time.

We checked if the system formed with (29) and (30) equations has solution at $t_{FRW} \simeq 0.524$ and found that it does, even that it is very complicated and dependent on the constants c_1 - c_3. A fine tuning of these constants will be possible by comparing the result of the 3D simulation with the observed heliospheric electric and magnetic field topology.

With a variation in distance from the solar surface to the helioshere limit ($c_2 \in (0,70]$), at a constant time (e.g, $t_{FRW} = 0.524$), we can get a snapshot of the electric and magnetic field topology into the heliosphere. For this, as initial value for the fields we take $B_{0r} = B_{0\zeta} = B_{0\xi} = 0$ and $E_{0r} = E_{0\zeta} = E_{0\xi}$, obtained at $t_{FRW} = 0.5$ from:

$$\frac{1}{\cosh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]} \simeq 10^{-38} \to 0$$

and

$$\exp\left\{-c_1\ \sinh\left[\arctan\left(\frac{1}{\tan t_{FRW}}\right)\right]\right\} \simeq \exp\left(-10^{38}\right) \to 0$$

References

A.S. Popescu, "Gravito-magnetic Heliospheric Surfaces", AIP Conference Proceedings Series: Fifty Years of Romanian Astrophysics, Ed. C. Dumitrache et al., ISBN 978-0-7354-0400-7 (2007).