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Equilibrium stability of a nonlinear structural switching system with actuator delay

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A mathematician, like a painter or a poet, is a maker of patterns
G. H. Hardy.

Abstract

In this paper, a Lyapunov–Krasovskii functional is used to obtain sufficient conditions of asymptotic stability for the equilibrium of a nonlinear feedback system with state-dependent uncontrolled switching, herein called structural switching, and with actuator delay. The solution of the problem is addressed in two steps. First, a predictive feedback method is used to compensate the actuator delay of the associated linearized system. Thus, the time-delayed control is replaced with a state delay, and the effect of the control appears in a non-homogeneous term in the linearized system. Second, a theorem of asymptotic stability of equilibrium is obtained for the nonlinear switched system, whose linearized components were considered separately in the first step. The result is also valid for certain problems of state-dependent controlled switching. The numerical application, done on a consecrated real world system, the electrohydraulic servomechanism, highlights real difficulties, which are usually avoided by academic constructs in which the results are sometimes illustrated on insignificant models, represented, for example, by 2×2 didactic matrices.

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1. Introduction

The two main concepts of this paper are the time delayed systems and the switched systems. Therefore, a short introduction regarding the two types of systems is useful.

The systems with delay are also called hereditary systems, systems with after effects or dead time, and systems with time lags as well [1]. The classical authors of the field speak about the class of retarded differential-difference equations [2], differential equations with deviating argument [3], or retarded functional differential equations [4]. In the book [5], an association is made between the linear partial differential equations, so-called distributed parameter systems and the linear differential equations with delay, so-called lumped parameter systems. Both types belong to the class of infinite dimensional systems, a property given by the infinite number of solutions that the characteristic polynomials of delay equations may have; the characteristic polynomials are now exponential polynomials or quasipolynomials. In an influential work [6], Pontryagin gives a fundamental theorem on the zeros of quasipolynomials, which opens a way to analyse the stabilization of linear time invariant systems with time delay [7]. Important books have appeared since the 1950s, such as [8], and continuing with [2–4, 9–14]. At first, the attention was drawn to the open loop systems with state delay. Later, the progress made in this field facilitates the development of the theory of systems with time delay on input (control) or output (measurement) [15–19].

An elegant definition of the switched systems is that from the paper [20]: “by a switched system, we mean a hybrid dynamical system consisting of a family of continuous-time subsystems and a rule that orchestrates the switching between them”. In book [21], the switched systems are presented from a theoretical perspective of the switching law synthesis to obtain stable switching systems. The used mathematical concepts and tools are single and multiple Lyapunov function, Lie-algebraic stability criteria, limited-rate switching no smaller than so-called dwell-time (i.e., the time between switchings [22]). A persistent dwell-time switching scheme is utilized in [23] to describe the successive occurrence of slow and fast switching in a switched singular system with sensors failures. The same persistent dwell-time switching strategy is employed to represent the switching among neural networks used for H_∞ filtering [24] or for state estimation in a problem of mixed $H_\infty/l_2 - l_\infty$ [25]. These are examples from which one can see how the boundaries of the concept of switched system can be extended to formulate and solve new problems. We mention that dwell time switching schemes are also studied for systems with variable time delay, on state or on control, since these schemes can optimize the dynamic behavior of the system by controlling the time delay.

It is important to note that the system considered in the application from Section 4, the mathematical model of the electrohydraulic servomechanism (EHS), does not belong to the above categories. EHS is the representative example for a system with state-dependent uncontrolled switching. This fact results from the constructive-functional scheme of EHS (Fig. 1), whose mathematical model (11) and (12) is broken down into two components that describe the servovalve ports opening on one side and the other of hydraulic null during dynamical functioning. Applying analysis tools such as Lyapunov–Malkin theory, geometric control and Common Quadratic Lyapunov Function [26,27], in the works [28–32] the equilibrium stability of this switching type system has been studied.

In this article we consider systems that are both switched and delayed. In some recent works, [33,34], stability criteria based on multiple Lyapunov–Krasovskii functionals are proposed for studying linear and nonlinear open-loop, in other words without control, state-delayed switched systems. The stabilization problem of switched control systems with delay

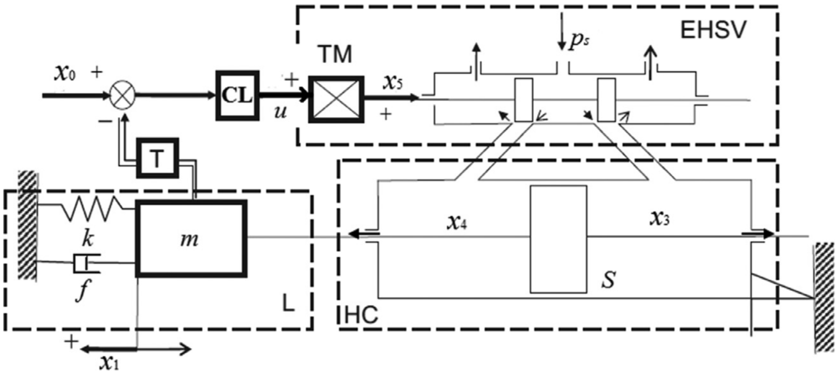

Fig. 1. Block diagram of the servovalve controlled EHS. HC: hydraulic cylinder with piston; L: load; C: controller; T: transducer; TM: torque motor; EHSV: electrohydraulic servovalve.

in switching law is studied in [35] by using the method of Lyapunov functions and delay inequalities. In [36], exponential stability analysis and stabilization for linear switched systems with time-varying delay are approached by using the method of interconnecting systems and applying the small gain theorem. Mathematical tool of Lyapunov–Krasovskii functionals was also used in work [37] to obtain appropriate conditions involving an average dwell time for the input-to-state stability of switched nonlinear systems with delayed input and in the presence of disturbances. In [38], a global exponential stabilization criterion is established by introducing restrictions on the linear part of the system in terms of Metzler matrices. In all the just quoted papers, the switching refers again to the derivation of a switching law that stabilizes the system.

The purpose of this paper is to propose a solution to the problem of the equilibrium stability for a complex nonlinear feedback system with control delay and uncontrolled state-dependent switching. We specify that this assumed problem had as starting point the challenges risen by the behavior of the mathematical model of EHS, one of the most popular mathematical models of automation: remembering Norbert Wiener's words "the present age is the age of servomechanisms" [39]. EHSs recently attracted attention in control community. At least a few works are to be mentioned [40–43], but also results like: the robust tracking control synthesis based on three compensators [44], the using of Lyapunov–Malkin paradigm [45] to handle the equilibrium stability critical case [46], the nonlinear geometric switching type control synthesis [47], the backstepping control synthesis [48], the equilibrium stability in servoelastic framework control [49], the synthesis for model parametric uncertainty [50] etc. Excepting paper [47], all these quoted papers, and many others, provide only partial solutions, to the extent that they disregard the nature of the EHS model as described by two components according to the sign of one of its state variables. Particular attention was paid to the hydraulic control systems starting from the Second World War when the mechanohydraulic servomechanisms equipped the military aircraft [51,52]. After 1949, they became parts of the mechanical engineering system of jet airlines for civil transport. A fundamental bibliography in the field should contain the references such as [53–55].

The solution of the problem stated above is addressed in a *first step* in Section 2. Namely, the stability of the linearized systems through Taylor series development is approached separately, eluding for the moment the switching structure. We specify that in the paper the delayed control variable is present in the nonlinear system through a linear term with constant matrix coefficient, which therefore does not require linearization. The predictive feedback method is used to compensate for delay in the linearized system with actuator delay. Thus, the delay on the actuator is replaced with a state delay, with the consequence of adding a non-homogeneous term that contains the effect of the applied control. The method of predictive feedback is applied in a different approach from some existing ones in the literature (see, e.g., [56]), but consistent with others (see, e.g., [57]). Given that the obtained state delay came from a delay in control, usually well defined by a chosen maximal constant value, in this paper it was not necessary to resort to the networked time delay concept [58,59].

In Section 3, in the *second step* of the problem solving, sufficient conditions for asymptotic stability of equilibrium are given for the extended nonlinear systems, this time considering nonlinear system components together, as a switched system in whole. A first main contribution of the paper, Theorems 2 and 3, are in fact a reassessment and extension of the results from paper [60] given for two open loop linear systems with state delay and controlled switching. What is important to note is that Theorem 3 presents a generalized result for m systems, also valid for the case of a state-dependent controlled switching. The checking and the criticism of the results is done in Section 4 by numerical simulations on a consecrated system, the mathematical model of the EHS. To the best of our knowledge, this application of the two Theorems on this mathematical model is a new and important extension of the works elaborated by the authors and represents a second main contribution of the paper. The study ends with concluding remarks.

2. A short review on systems with delay. Predictive feedback control design

This Section begins with some basic definitions regarding the Lyapunov stability for nonlinear systems with time delay. A common strategy in control theory is to synthesise the control law on the linearized system and then to evaluate the stability and robustness of the nonlinear system in the closed loop thus obtained. Here we have a delayed control, which will be addressed with the predictive feedback method. Thus, a closed loop system like system (1) or system (8) is obtained, hence it is legitimate to refer to the definitions of stability to the system (1). We mention that there is another synthesis method in the case of systems with delayed control, the reduction method [19], which finally gives a closed loop system similar to the one obtained through the predictive feedback method.

At the end of the Section, the Krasovskii general theorem of stability for nonlinear systems with state delay is presented, which will be extended in Section 3 to switched systems with state delay.

2.1. Basic definitions related to Lyapunov stability of nonlinear systems with delay

Suppose that $\boldsymbol{f} : D \times D \to \Re^n$ is a locally Lipschitz and continuous function defined on Cartesian products of domains $D \subset \Re^n$ into $\Re^n$, and consider the nonlinear differential system with delay

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{x}(t-h)) \tag{1}$$

This expression is obtained even if one starts from a nonlinear system with delay on the actuator and then the state feedback loop is closed, as it will be done in Sections 3 and 4. Let us note by $\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})$ the solution of system (1) with initial condition $\boldsymbol{x}(\theta; t_0, \boldsymbol{\varphi}) := \boldsymbol{\varphi}(\theta), \theta \in [t_0 - h, t_0]$, often written as $\boldsymbol{x}(t_0 + \theta) = \boldsymbol{\varphi}(\theta), \theta \in [t_0 - h, t_0]$. The initial conditions refer therefore to initial time $t_0 \geq 0$ and an initial function $\boldsymbol{\varphi} : [t_0 - h, t_0] \to \Re^n$, belonging to a certain functional space, e.g., to the normed space of continuous functions $C([t_0 - h, t_0], \Re^n)$ with the Euclidean norm $\|\boldsymbol{\varphi}\|_h = \sup_{t_0-h\leq\theta\leq t_0}\|\boldsymbol{\varphi}(\theta)\|$. The solution exists and is unique in well-defined conditions (see, for more details, [13], §1.2. Th. 1.1). If the practical method of building the solution is the step-by-step method, then it is clear that in order to advance one step it is necessary to know the solution along the previous step. Let's introduce one of the attributes of system (1), *the state* $\boldsymbol{x}_t(t_0, \boldsymbol{\varphi}) := x(t+\theta), \theta \in [t_0 - h, t_0]$ *at a time instant* $t \geq t_0$ *along a solution* $\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})$, defined as the restriction of this solution on the time interval $[t - h, t]$. If there is no risk of confusion, the arguments t_0 (usually, 0) and $\boldsymbol{\varphi}$ can be omitted, thus writing $\boldsymbol{x}(t)$ instead of $\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})$ and $\boldsymbol{x}_t$ instead of $\boldsymbol{x}_t(t_0, \boldsymbol{\varphi})$. In fact, system (1) is often written as follows

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}_t), \boldsymbol{x}_t \in C([-h, 0], \Re^n) \tag{1a}$$

As in the consecrated approach to the Lyapunov's stability theory, a certain equilibrium point of system (1a), if there is one, will be previously translated to zero. A perturbation of this zero equilibrium is introduced as $\boldsymbol{x}(t_0, \boldsymbol{\varphi}) = \boldsymbol{\varphi}(0) = \boldsymbol{x}_0 \neq 0$. Some definitions for what is commonly known as stability in the sense of Lyapunov will be briefly presented from well-known sources such as [8,13], as well as [61–64]. Obviously, the concept of instability simply results from the denial of the definition of stability.

Definition 1 ([8], apud [13], Definition 1.2)**.** The zero equilibrium (or zero solution) of Eq. (1a) is said to be stable if for any $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that for every initial condition $\boldsymbol{x}_0$ and function-condition $\boldsymbol{\varphi} \in C$ with $\|\boldsymbol{x}_0\| \leq \|\boldsymbol{\varphi}\|_h < \delta(\varepsilon)$, the inequality $\|\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})\| < \varepsilon$ holds for $t \geq 0$.

Unlike the cited sources, where the definition was given for non-autonomous systems, we transcribed the definition in the case of autonomous system (1a). A second, non-essential, difference appears by the presence of the initial condition, which explains how to introduce the disturbance of the equilibrium state, see the simulations in Section 4. For non-autonomous systems, like system (1), there may be an additional dependence of δ, $\delta = \delta(\varepsilon, t_0)$. If δ can be chosen independently of t_0, then the zero solution would have been called uniformly stable. For autonomous systems the stability and uniform stability coincide, since the change of t_0 only returns to a corresponding translation of the solution over time. In other words, the value $\delta(\varepsilon)$ is always smaller than or equal to ε [13], and the above-defined stability is a *weak* one. A *stronger* stability is introduced further.

Definition 2 ([13], Definition 1.3, [63], Definition 5.6)**.** The zero solution of Eq. (1a) is said to be asymptotically stable if (a) it is stable and (b) it is an attractor, i.e., once $\delta(\varepsilon) > 0$ is chosen such that $\|\boldsymbol{x}_0\| \leq \|\boldsymbol{\varphi}\|_h < \delta(\varepsilon)$, then $x(t, t_0, \boldsymbol{\varphi}) \to 0$ as $(t - t_0) \to \infty$. The set of $\boldsymbol{x}_0$ points that meet the previous condition is called the basin of zero solution.

Definition 3 ([13], Definition 1.4)**.** The zero solution of system (1′) is said to be exponentially stable if there exist $\Delta_0 > 0$, $\sigma > 0$, and $\gamma \geq 1$ such that for every $t_0 \geq 0$ and any initial function $\boldsymbol{\varphi} \in PC([-h,\ 0], R^n)$, with $\|\varphi\|_h < \Delta_0$, the following inequality holds: $\|x(t,\ t_0,\ \varphi)\| \leq \gamma\|\varphi\|_h e^{-\sigma(t-t_0)},\ t \geq t_0$.

Obviously, the exponential stability is *even stronger* than the asymptotic one, given the fastest decrease to zero of the perturbed solution, the exponential decrease.

The types of stability defined so far refer to *local* stability, in the sense that they describe the behavior of the solutions that have as a start in a limited neighbourhood of the equilibrium point. The zero solution of system (1a) is *globally asymptotically stable* if it is stable and if it is a global attractor. The zero solution is a *global attractor* if its basin is equal to $\Re^n$.

We have recalled these definitions to mark the fact that in any system of the real world, as is the system discussed in Section 4, there are geometric, constructive and functional restrictions, which are reflected in the size of the states. Therefore, for dynamic real world systems, especially for the feedback ones, it is difficult, if not impossible, to speak about something different than the local stability, in the sense of the above definitions.

2.2. The predictive feedback synthesis method for linear systems with actuator delay

Consider the basic linear time invariant system

$$\begin{array}{l}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}_c u(t-h), \boldsymbol{A} \in \Re^{n\times n}, \boldsymbol{B}_c \in \Re^{n\times 1} \\ u(t) = u_0(t), \quad -h \le t \le 0, \quad h > 0, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \ne 0\end{array} \tag{2}$$

where $(\boldsymbol{A}, \boldsymbol{B}_c)$ is a complete controllable pair [18,65]. The objective is to study the zero equilibrium of the system for $t > 0$, given a state feedback control $u(x(t))$ and initial conditions $u_0(.)$, x_0; the last one is introduced as a perturbation of the zero equilibrium in Eq. (2). If the delay h is not too large, a control law obtained, for instance, by the finite-dimensional LQR (Linear Quadratic Regulator) algorithm [65,66] has a certain robustness, therefore the stability of the system is preserved [67]. In the case of a relatively large delay h, the stability is no longer guaranteed. In paper [66], the synthesis of the control was extended within the LQG (Linear Quadratic Gaussian) framework, having as a guide work [68].

The first solution to increase robustness in the presence of delays, in the frequency domain and for single-input-single-output systems stable in open-loop, was proposed in the late 1950s, namely the well-known Smith predictor [15]. The method was fruitful but also controversial for shortcomings concerning the robustness. Indeed, a very recent work recommends to “forget the Smith predictor”, and use instead a well-tuned PI (Proportional-Integral) or PID (Proportional–Integral–Derivative) controller [69]. Throughout the ’60s–’80s, new approaches related to system (2) have emerged, e.g., the finite spectrum assignment method [17] and the Artstein–Kwon–Pierson reduction method [18,19]. The two related methods ultimately led to the predictive feedback control method even if they started from different ideas. A remark from [57] is worth remembering: against appearances, system (2) has “a finite dimensional flavor”.

The predictive feedback method may be summarized as follows. The method is based on the assumption that system (2) in the absence of the delay can be stabilized with a feedback control $u(t) = \boldsymbol{Kx}(t)$. The objective is to find a feedback control law in the presence of the delay such that $u(t-h) = \boldsymbol{Kx}(t)$, what can be written as $u(t) = \boldsymbol{Kx}(t+h)$ (which appears as no implementable!), so a state predictor is previously necessary.

Proposition 1. *Consider system (2) with $(\boldsymbol{A}, \boldsymbol{B}_c)$ a controllable, or at least stabilizable pair. $\boldsymbol{A}$ may be even unstable. By considering a state predictor*

$$\boldsymbol{x}_p(t) := \boldsymbol{x}(t+h) = e^{\boldsymbol{A}h}\boldsymbol{x}(t) + \int_{-h}^{0} e^{-\boldsymbol{A}s}\boldsymbol{B}_c u(t+s)ds \tag{3}$$

the system with control delay Eq. (2) can be replaced with the following non-homogeneous system with state delay

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{A}_d\boldsymbol{x}(t-h) + \boldsymbol{B}_c\boldsymbol{K}\int_{-h}^{0} e^{-\boldsymbol{A}s}\boldsymbol{B}_c u(t+s-h)ds, \boldsymbol{A}_d := \boldsymbol{B}_c\boldsymbol{K}e^{\boldsymbol{A}h}. \tag{4}$$

Proof. Relationship Eq. (3) is easy to check by making the derivative and taking into account that (see Eq. (2))

$$\dot{\boldsymbol{x}}(t+h) = \boldsymbol{A}x(t+h) + \boldsymbol{B}_c u(t). \tag{5}$$

Now, calculate a feedback gain $\boldsymbol{K}$ for system (2) (based for example on the LQR algorithm [65]), as if the system was without delay. Introducing the feedback predictive control

$$u(t) = \boldsymbol{K}\boldsymbol{x}(t+h) = \boldsymbol{K}\left(e^{\boldsymbol{A}h}\boldsymbol{x}(t) + \int_{-h}^{0} e^{-\boldsymbol{A}s}\boldsymbol{B}_c u(t+s)ds\right) \tag{6}$$

and substituting the control variable $u(t-h)$ in Eq. (2) lead to system (4). □

It is not without interest to make two Remarks.

Remark 1. The non-homogeneous part of linear system (4) is an integral term that naturally contains the effect of the control variable

$$\boldsymbol{B}_c\boldsymbol{K}\int_{-h}^{0} e^{-\boldsymbol{A}s}\boldsymbol{B}_c u(t+s-h)ds. \tag{7}$$

It can be easily to check that the control variable in Eq. (7) is causal and consequently the control law to the actuator is implementable. Indeed, the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}_c u(t-h)$, with initial conditions $u_0(.)$, $\boldsymbol{x}_0$, can be integrated so giving the solution $\boldsymbol{x}(t)$ on the interval $[0,\ h]$. Thus $u(t) = \boldsymbol{K}\boldsymbol{x}(t)$, $t \in [0,\ h]$, is known. Further, on the interval $[h,\ 2h]$, it is necessary to know $u(t)$ on the interval $[-h,\ h]$ translated one step h back. Indeed, the control variable argument varies on Cartesian product $[h,\ 2h] \times [-h,\ 0]$ of t and s variation, respectively. When referring to the lower limit h of the t variation, then the argument of u varies in the range $[-h,\ 0]$. When referring to the upper limit 2h of the t variation, then the argument of u varies in the range $[0,\ h]$. The same reasoning is repeated for each interval. This approach of finding a solution with an initial value problem Eq. (2) is known as the step by step method [2].

Remark 2. The discretization of the non-homogeneous term, necessary for the on-line implementation of control, could destabilize the closed loop system [70].

2.3. A classical stability theorem for zero solution of nonlinear systems with state delay

Let $V : C \to \Re$ be continuous and $\boldsymbol{x}_t$ be the solution of Eq. (1a), generated in the presence of a disturbance of the equilibrium state. The upper right-hand derivative of $V(\boldsymbol{x}_t)$ along the solution of Eq. (1a) is defined by $\dot{V}(\boldsymbol{x}_t) = \lim\ \sup_{s\to 0^+} \frac{1}{s}[V(\boldsymbol{x}_{t+s}) - V(\boldsymbol{x}_t)]$.

Theorem 1. *Consider the system defined by Eq. (1a). Suppose that* $\boldsymbol{f} : C \to \Re^n$ *maps every bounded set of* C *into a bounded set in* $\Re^n$ *and that* $\alpha,\ \beta,\ \psi : [0,\infty) \to [0,\infty)$ *are continuous nondecreasing functions, and* $\alpha(0) = \beta(0) = 0$. *A sufficient condition of asymptotic stability of the zero solution of Eq. (1a) is the existence of a continuous differentiable function* $V : C \to \Re$ *with the properties* 1) $\alpha(\|\boldsymbol{x}(t)\|) \le V(\boldsymbol{x}_t) \le \beta(\|\boldsymbol{x}_t\|_h)$ *and* 2) $\dot{V}(\boldsymbol{x}_t) \le -\psi(\|\boldsymbol{x}(t)\|)$.

Most of the consulted sources [1,7,13,71] say that Theorem 1, often called Lyapunov–Krasovskii Stability Theorem, originated in Krasovskii's book [8], see Theorems 31.1–31.3. The above-mentioned version is close to that of [7] and [60], the latter quoting as source [4].

A functional $V(\boldsymbol{x}_t)$ satisfying the conditions of Theorem 1 is called a Lyapunov–Krasovskii functional [13,60]. Thus, the Lyapunov function, dependent on state $\boldsymbol{x}(t)$, is replaced by a functional dependent on the "true" state $\boldsymbol{x}_t$ [13].

3. A stability theorem for nonlinear structural switching system with actuator delay

In Section 2, it was shown that, by applying a predictive feedback control based on a LQR synthesis, the linear system with actuator delay Eq. (2) can be reduced to a linear system with state delay Eq. (4), and with a non-homogeneous term Eq. (7) containing the effect of the control variable. The following nonlinear feedback system with structural switching and delay transferred from control to state is now considered:

$$\begin{aligned}&\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{di}\boldsymbol{x}(t-h) + \boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^{0} e^{-\boldsymbol{A}_i s}\boldsymbol{B}_c u_i(t+s-h)ds + \boldsymbol{F}_i[\boldsymbol{x}(t)],\\ &\boldsymbol{A}_{di} := \boldsymbol{B}_c\boldsymbol{K}_i e^{\boldsymbol{A}_i h},\ i = 1, \ldots, m.\end{aligned} \tag{8}$$

System (8) is nonlinear switching extension of system (4), in which are added the remainders of order one $\boldsymbol{F}_i(\boldsymbol{x}(t))$ from the Taylor series development around the origin. The evaluation of such terms is not a simple matter, especially in the case of mathematical models characterizing real world, as in Section 4. To analyse a switched system like Eq. (8), one more key-Assumption (A3) must be added to Assumptions 1 and 2 of Theorem 1:

- (A_1) *There exist the continuous positive increasing functions* $\alpha,\ \beta : [0,\infty) \to [0,\infty)$, $\alpha(0) = \beta(0) = 0$ *and the continuous functions* $V_i(x_t) : C \to \Re$ *such that* $\alpha(\|\boldsymbol{x}(t)\|) \le V_i(\boldsymbol{x}_t) \le \beta(\|\boldsymbol{x}_t\|_h)$, $i = 1, \ldots, m$.
- (A_2) *For each* $i = 1, \ldots, m$ *in Eq. (8), there are the continuous increasing functions* $\psi_i(\cdot)$ *with the properties* $\psi_i(s) > s$ for $s > 0$, $\psi_i(0) = 0$ *and* $\dot{V}_i(\boldsymbol{x}_t) \le -\psi_i(\|\boldsymbol{x}(t)\|)$, *wherein the derivative is considered along the system trajectories.*
- (A_3) *There is* $\mu > 1$ *such that* $V_i(\boldsymbol{x}_t) \le \mu V_j(\boldsymbol{x}_t)$ for all $\boldsymbol{x}_t \in C([-h,\ 0],\ \Re^n)$ and for $i \neq j$.

The equilibrium stability of system (8) will be addressed using an extension of the functional from [60], and, in accordance with Section 2.3, we will also call it a Lyapunov–Krasovskii functional. The following theorem shows the conditions under which the proposed functional fulfils (A_1) and (A_2).

Theorem 2. *Consider system (8) with: (a)* $\boldsymbol{A}_i$*− Hurwitz matrices, therefore there are symmetric positive definite matrices* $\boldsymbol{P}_i$ *satisfying Lyapunov matrix equations* $\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i = -\boldsymbol{Q}_i$ *for some symmetric positive definite matrices* $\boldsymbol{Q}_i$ *and (b)* $\boldsymbol{A}_{di}$*− small enough matrices, specifically* $\|\boldsymbol{P}_i\boldsymbol{A}_{di}\| < \lambda_{\min}(\boldsymbol{Q}_i)/2$ *, therefore there are* $\omega_i > 0$ *such that* $\|\boldsymbol{P}_i\boldsymbol{A}_{di}\| \le \omega_i < \lambda_{\min}(\boldsymbol{Q}_i)/2$*. For each* $i = 1, \ldots, m$ *in Eq. (8), the following Lyapunov–Krasovskii functional is considered*

$$V_i(\boldsymbol{x}_t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t) + \omega_i\int_{t-h}^{t}\|\boldsymbol{x}(s)\|^2 ds \tag{9}$$

where $\omega_i > 0$*. Then* (A_1), (A_2) *are fulfilled for all* $i = 1, \ldots, m$*, as long as the functions* $\psi_i(\|\boldsymbol{x}(t)\|) := \{\lambda_{\min}(\boldsymbol{Q}_i) - 2[\omega_i + \lambda_{\max}(\boldsymbol{P}_i)(M_i\|\boldsymbol{x}\| + N_i)]\}\|\boldsymbol{x}(t)\|^2$ *are positive.*

Proof. It is easy to see that (A_1) is fulfilled if one choose $\alpha(\|\boldsymbol{x}(t)\|) = \min_i(\lambda_{\min}(\boldsymbol{P}_i))\|\boldsymbol{x}(t)\|^2$ and $\beta(\|\boldsymbol{x}_t\|_h) = \max_i(\lambda_{\max}(\boldsymbol{P}_i) + \omega_i h)\|\boldsymbol{x}_t\|_h^2$ [60]. A calculation not very complicated, see below, establishes the expressions of functions $\psi_i(\|\boldsymbol{x}(t)\|)$ and, consequently, the conditions that the system parameters have to fulfil in order that Assumption (A_2) holds. In the following, the control variable will be substituted based on the state value, $u_i(t) = \boldsymbol{K}_i\boldsymbol{x}(t)$). Also, we will be interested in a formula defining an upper bound of $\boldsymbol{F}_i[\boldsymbol{x}(t)]$ terms in Eq. (8). For each fixed i, consider a fixed component $f_{i,l}$ of the vector function $\boldsymbol{f}_i$ in Eq. (1) (we indexed with i the vector function $\boldsymbol{f}$, since we take the structure from Eq. (1) for switched systems). A such upper bound is to be found, taking into account the Lagrange form of theR_1 remainder of order one in Taylor series development of the component $f_{i,l}(\boldsymbol{x})$, $(\boldsymbol{F}_i[\boldsymbol{x}(t)])_l := R_1 = \frac{1}{2}\sum_{j,k=1}^{n}\frac{\partial^2 f_{i,l}}{\partial x_j \partial x_k}(\boldsymbol{b})x_j x_k$ (with $()_l$ was noted the component l of the vector in parentheses). R_1 exists if the function$f_{i,l}(\boldsymbol{x})$ is twice differentiable in $\boldsymbol{b}$. Moreover, $R_1(\boldsymbol{x}) = o(|x|)$ as $\boldsymbol{x} \to 0$. In compact vector form, each $(\boldsymbol{F}_i[\boldsymbol{x}(t)])_l$ term is written as $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{b})\boldsymbol{x}/2$, and includes the Hessian matrix $\boldsymbol{H}$ calculated in a point $\boldsymbol{b} = t^*\boldsymbol{x}, t^* \in (0, 1)$. Because it is usually very difficult to find exactly this $t^* \in (0, 1)$, instead we will look for numbers $M_i > 0$ that we know they exist, such that $\|\boldsymbol{F}_i[\boldsymbol{x}(t)]\| \le M_i(\|\boldsymbol{x}\|^2)$. Therefore, we will calculate:

$$
\begin{aligned}
\dot{V}_i(\boldsymbol{x}_t) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{x}}(t) + \omega_i\big(\|\boldsymbol{x}(t)\|^2 - \|\boldsymbol{x}(t-h)\|^2\big) \\
&= \left(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{A}_{di}^{\mathrm{T}} + \boldsymbol{F}_i^{\mathrm{T}}[\boldsymbol{x}(t)] + \left[\boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^{0} e^{-\boldsymbol{A}_i s}\boldsymbol{B}_c u(t+s-h)ds\right]^{\mathrm{T}}\right)\boldsymbol{P}_i\boldsymbol{x}(t) \\
&\quad + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}_i\left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{di}\boldsymbol{x}(t-h) + \boldsymbol{F}_i[\boldsymbol{x}(t)] + \left[\boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^{0} e^{-\boldsymbol{A}s}\boldsymbol{B}_c u(t+s-h)ds\right]\right) \\
&\quad + \omega_i\big(\|\boldsymbol{x}(t)\|^2 - \|\boldsymbol{x}(t-h)\|^2\big) \\
&= \boldsymbol{x}^{\mathrm{T}}(t)\big(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i\big)\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{A}_{di}\boldsymbol{x}(t-h) \\
&\quad + \omega_i\big(\|\boldsymbol{x}(t)\|^2 - \|\boldsymbol{x}(t-h)\|^2\big) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{F}_i[\boldsymbol{x}(t)] \\
&\quad + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\left[\boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^{0} e^{-\boldsymbol{A}_i s}\boldsymbol{B}_c u(t+s-h)ds\right] \\
&\le -\lambda_{\min}(\boldsymbol{Q}_i)\|\boldsymbol{x}(t)\|^2 + \omega_i\big(\|\boldsymbol{x}(t)\|^2 + \|\boldsymbol{x}(t-h)\|^2\big) \\
&\quad + \omega_i\big(\|\boldsymbol{x}(t)\|^2 - \|\boldsymbol{x}(t-h)\|^2\big) + 2M_i\lambda_{\max}(\boldsymbol{P}_i)\|\boldsymbol{x}(t)\|^3 \\
&\quad + 2h\lambda_{\max}(\boldsymbol{P}_i)\lambda_{\max}(\boldsymbol{B}_c\boldsymbol{K}_i)^2 \max_{-h\le s\le 0}\left\|e^{\boldsymbol{A}_i s}\right\|\|\boldsymbol{x}(t)\|^2 \\
&\le -\{\lambda_{\min}(\boldsymbol{Q}_i) - 2[\omega_i + \lambda_{\max}(\boldsymbol{P}_i)(M_i\|\boldsymbol{x}(t)\| + N_i)]\}\|\boldsymbol{x}(t)\|^2
\end{aligned}
$$

which ends the proof. □

In order to demonstrate the stability theorem, two more Propositions are needed.

Proposition 2. *If the assumption (A_2) is fulfilled, then for any pair of consecutive switching times $\{t_p, t_q\}$ of the* ith *component of system* (8) *with $t_p < t_q$ and with i^{th} component system active at t_p and t_q, respectively, there are the constants $0 < \xi_i < 1,\ i = 1, ..., m$ such that*

$$
V_i\big(\boldsymbol{x}_{t_q}\big) - V_i\big(\boldsymbol{x}_{t_p}\big) \le -\xi_i V_i\big(\boldsymbol{x}_{t_p}\big) \tag{10}
$$

Proof. Once the equilibrium $\boldsymbol{x} = 0$ of the dynamic autonomous system (8) is perturbed, it will evolve as a switched system, but so that $V_i(\boldsymbol{x}_t)$ are strictly decreasing, since $\dot{V}_i(\boldsymbol{x}_t) < 0$, excepting $\boldsymbol{x} = 0$. An other presumptive equilibrium point is $\boldsymbol{x}_e$ with $\|\boldsymbol{x}_e\| = [\lambda_{\min}(\boldsymbol{Q}_i) - 2(\omega_i + \lambda_{\max}(\boldsymbol{P}_i)N_i)]/2\lambda_{\max}(\boldsymbol{P}_i)M_i$, but this is reached only as time tends to infinity. Then $V_i(\boldsymbol{x}_{t_q}) - V_i(\boldsymbol{x}_{t_p}) < 0$, so there is $0 < \xi_i < 1$, such that $V_i(\boldsymbol{x}_{t_q}) - V_i(\boldsymbol{x}_{t_p}) \leq -\xi_i V_i(\boldsymbol{x}_{t_p})$. □

The last inequality would only be contradicted by the cancellation of the "energy" derivative along a whole trajectory interval, and not just within a point. Either, this would mean that the total energy of a moving system remains constant, as for example in the case of a stable limit cycle, generated by essential nonlinearities such as dead zone, saturation etc. We exclude this inconvenience in this paper.

Proposition 3. *Lyapunov–Krasovskii functional Eq. (9) fulfils Assumption (A_3).*

Proof. For compliance, we note that the demonstration is given in detail in [60] and is based on the choice.

$$\mu = \max\left\{\sup_{i,j}\frac{\lambda_{\max}(\boldsymbol{P}_i)}{\lambda_{\min}(\boldsymbol{P}_j)}, \sup_{i,j}\frac{\omega_i}{\omega_j}\right\}$$

□

Theorem 3. *A sufficient condition of asymptotic stability of the zero solution of system* (8) *is that the functionals $V_i(\boldsymbol{x}_t)$, $i = 1, \ldots, m$ given by Eq. (9) meet assumptions (A_1), (A_2).*

The result given in [60] is performed in the simplified case $m = 2$, perfectly suited for the application in Section 4, in which a switching called structural, as defined by the mathematical model of EHS, was considered. In the following, we will give a general result, applicable to a system with multiple switches, realized either structurally, as in the case of EHS, or by control. This is for example the case of the gain scheduling control of an airplane, in which the flight mission is performed with a number of controllers associated with a number of linearized components of the nonlinear mathematical model depending on the flight height and speed. Therefore, the approach from [60] will be extended to the case of m systems of type Eq. (8), also clarifying some of the statements there.

Proof. Thus, consider Lyapunov functionals $V_1(\boldsymbol{x}_t), \ldots, V_n(\boldsymbol{x}_t)$ for the switching system (8) satisfying conditions (A_1), (A_2). For technical reasons, without affecting the generality, since the succession of the switches changes from case to case (in the given example of the flight mission), let's admit that after $m+2$ switches the system went through all the m configurations, which we would call modes; such a succession could 1, 4, 3, 2, 3, 4, 1, 5, 6,…,m From (A_1) and based on the continuity of increasing functions $\alpha(\cdot)$ and $\beta(\cdot)$, $\alpha \leq \beta$, it is obvious that for any $\varepsilon > 0$ there exist $\delta(\varepsilon) > 0$ and $\mu > 1$ such that $\beta(\delta) = \alpha(\varepsilon)/\mu^{m+2}$. Now, assume that the subsystem with $i = 1$ is active on $[t_{1,0}, t_{1,1})$ and $\|\boldsymbol{x}(t_0 + \theta)\| = \|\varphi(\theta)\| \leq \delta$ where $t_{1,0}$ is an initial time and $\theta \in [-h, 0]$. This inequality in norm is ensured by the fact that δ and μ can be conveniently chosen. According to (A_2), $\dot{V}_1 < 0$ on $[t_{1,0}, t_{1,1})$, thus $V_1(\boldsymbol{x}_{t_{1,1}}) < V_1(\boldsymbol{x}_{t_{1,0}}) \leq \beta(\delta) = \alpha(\varepsilon)/\mu^{m+2}$, for $t \in [t_{1,0}, t_{1,1})$. Then, we assume that at time $t_{1,1}$, the subsystem with mode $i = 1$, active on $[t_{1,0}, t_{1,1})$, switches (as a result of an inherent dynamic change of sign of a variable x_q, $q \in \{1 \div n\}$, to the subsystem with $i = 4$ (as in the example given above), which will be active on $[t_{4,0}, t_{4,1})$. Thus, taking into account the continuity of functional V_1 in $t_{4,0} := t_{1,1}$ and according to (A_3), we have $V_4(\boldsymbol{x}_{t_{4,0}}) \leq \mu V_1(\boldsymbol{x}_{t_{4,0}}) < \mu\alpha(\varepsilon)/\mu^{m+2} = \alpha(\varepsilon)/\mu^{m+1}$. After another $m+1$ switches as in the example given above 4, 3, 2, 3, 4, 1, 5, 6,…,m, and based on the same Assumption (A_2) and on relation (10), the following sequence of inequalities is obtained: $V_1(x_t) < V_1(x_{t_{1,k}}) < V_1(x_{t_{1,0}}) < \alpha(\varepsilon)$, $V_2(x_t) < V_2(x_{t_{2,k}}) < V_1(x_{t_{2,0}}) < \alpha(\varepsilon)$,…,$V_m(x_t) < V_m(x_{t_{m,k}}) < V_1(x_{t_{m,0}}) < \alpha(\varepsilon)$, valid, respectively, on the overlapped intervals $t \in [t_{1,k}, t_{1,k+1}), \ldots, t \in [t_{m,k}, t_{m,k+1}), k = 0, 1, \ldots$. Therefore, putting together the above mentioned relationships, it results $V_i(\boldsymbol{x}_t) \le \alpha(\varepsilon)$ for $t \ge t_{1,0}$, $i = 1, \ldots, m$. Thus, for now, it has been shown the boundedness of Lyapunov–Krasovskii functionals, so it was shown that the "energy" of the system perturbed by initial conditions does not increase. But we will show that this energy, measured by the Lyapunov–Krasovski functional, decreases indefinitely as time increases. It is easy to see that, according to relation (10), $V_1(\boldsymbol{x}_{t_{1,k}}) \le (1-\xi_1)^k V_1(\boldsymbol{x}_{t_0}) < (1-\xi_1)^k \alpha(\varepsilon) < \alpha(\eta)$ for $t_{1,k} \ge t_M, \ldots, V_m(\boldsymbol{x}_{t_{m,k}}) \le (1-\xi_m)^k V_m(\boldsymbol{x}_{t_{m,0}}) < (1-\xi_2)^k \alpha(\varepsilon) < \alpha(\eta)$ for $t_{m,k} \ge t_M$, where M is evaluated from inequality $M > \max\limits_{i=1,\ldots m} (1\mathrm{n}(\alpha(\eta)) - 1\mathrm{n}(\alpha(\varepsilon)))/1\mathrm{n}(1-\xi_i)$, $\eta \in (0, \varepsilon)$. The logarithm has been applied in each of the last inequalities for $V_1(\boldsymbol{x}_{t_{1,k}}),\ldots,V_m(\boldsymbol{x}_{t_{m,k}})$, respectively. There is yet another step to make, and the demonstration is over: based on (A_2), $V_m(\boldsymbol{x}_t) < V_m(\boldsymbol{x}_{t_{m,k}})$ for $t \in [t_{m,k}, t_{m,k+1})$ and choosing $T \ge t_M - t_{1,0}$, it results what we intended to prove, namely $V_i(\boldsymbol{x}_t) < \alpha(\eta)$forall $t \ge t_0 + T$. In other words, with the restrictions stipulated in Theorem 2, the equilibrium of system (8) is asymptotically stable, the energy of the perturbed system, measured by the Lyapunov-Krasovski functional, vanishing over time. □

The principle of equilibrium stability as a consequence of the evolution of the total energy of a system to a minimum originates in the works of the Italian mathematician and physicist Torricelli [63]. In the following, the results from this Section are applied to study the stability of the EHS model with structural switching and delayed control.

4. Evaluation of conservativeness of results given in Theorem 3 for mathematical model of EHS

After the World War II, EHSs gained significant spreads and they became the right choice for a variety of areas: civil engineering, machine tools, mobile equipment and robots, radar antenna, land vehicles, naval and aerospace systems, missile launchers [52]. Thus, the study of stabilization and tracking problems for EHS is always attractive and important. EHS do not only allow the generation of large forces, but, thanks to modern control technology and sensors, are also capable of assuming important control tasks as, for example, highly precise positioning of heavy loads. The basic servovalve controlled EHS is a combination between an electrohydraulic servovalve (EHSV) and a hydrocylinder. EHS is itself an actuator with feedback, but it can be viewed broadly as a system with feedback of the real world, and the delay on control will be regarded as a delay of the control signal in the block of the EHSV. The mathematical model of EHS is a strongly nonlinear one, and reveals a switching nonlinearity due to constructive directional changes in the spool valve ports opening [47,72]. This physical aspect gives the mathematical model the statute of a system with structural, autonomous switching.

We will apply the results obtained in Section 3 to a mathematical model of EHS. The mathematical model below, inspired by paper [73], is the only one which satisfies the condition that the Jacobian matrices are Hurwitz matrices, as required by Theorems 2 and 3, among so many other studied models [28,31,47,48,50] with five states or with structural switching, which do not satisfy this condition. This mathematical model represents the splitting of the EHS system into two subsystems corresponding to $x_5 > 0$ and $x_5 < 0$, respectively.

$$\dot{x}_1 = x_2; \;\; \dot{x}_2 = (-kx_1 - fx_2 + Sx_3 - Sx_4)/m$$
$$\dot{x}_3 = B\big(Cx_5\sqrt{p_s - x_3} - Sx_2 + k_l(p_s - 2x_3)\big)/(V_0 + Sx_1)$$

$$\begin{aligned}
\dot{x}_4 &= B\big(-Cx_5\sqrt{x_4} + Sx_2 + k_l(p_s - 2x_4)\big)/(V_0 - Sx_1) \\
\dot{x}_5 &= (-x_5 + k_{SV}u_1(x(t-h)))/\tau_{SV}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{x}_1 &= x_2;\ \dot{x}_2 = (-kx_1 - fx_2 + Sx_3 - Sx_4)/m \\
\dot{x}_3 &= B\big(Cx_5\sqrt{x_3} - Sx_2 + k_l(p_s - 2x_3)\big)/(V_0 + Sx_1) \\
\dot{x}_4 &= B\big(-Cx_5\sqrt{p_s - x_4} + Sx_2 + k_l(p_s - 2x_4)\big)/(V_0 - Sx_1) \\
\dot{x}_5 &= (-x_5 + k_{SV}u_2(x(t-h)))/\tau_{SV},\ C := c_d w\sqrt{2/\rho}.
\end{aligned} \tag{12}$$

The initial conditions are $u_i(t) = u_{0,i}(t),\ -h \le t \le 0,\ h > 0,\ \boldsymbol{x}_i(0) = \boldsymbol{x}_{0,i} \ne 0, i = 1, 2$. The magnitude of the perturbation $\boldsymbol{x}(t_0, \boldsymbol{\varphi}_i) = \boldsymbol{\varphi}_i(0) = \boldsymbol{x}_{0,i} \ne 0$ is conditioned by the norm $\|\boldsymbol{\varphi}_i\|_h = \sup_{t_0-h\le\theta\le t_0}\|\boldsymbol{\varphi}_i(\theta)\|$. For convenience, the vector functions $\boldsymbol{\varphi}_i$ are taken as constants. The difference of models (11) and (12) with respect to the models indicated above consists in the presence of both leakages, internal, in spool valve of EHSV, and external, in hydrocylinder. The tank pressure, near of atmospheric pressure, is neglected. In the absence of any leakage in the physical model, the Jacobian matrices have two critically zeros, and a model such as that assumed in [55] with external and internal leakages, has singularity in the case of $x_1 = 0$. The notations in Eqs. (11) and (12) refer to the variables, parameters, and constants partially described in Fig. 1: x_1 is the load displacement, x_2 is the load velocity, x_3, x_4 are the pressures in the hydraulic cylinder chambers, x_5 is the EHSV spool valve opening and u is the control variable, an input voltage; p_s is the supply pressure; m is the equivalent inertial load of primary control surface reduced to the actuator rod; f is the combined coefficient of the damping and viscous friction forces on the load and the cylinder rod; k is an equivalent aerodynamic elastic force coefficient; k_l is the cumulative coefficient of leakages, see above; S is the effective area of the piston; V_0 is the cylinder semivolume; B is the bulk modulus of hydraulic oil;τ_{SV} is the servovalve time constant; k_{SV} is a coefficient of proportionality between the servovalve voltage and the displacement of the servovalve spool; c_d is the discharge coefficient in the servovalve spool; w is the valve port's width; ρ is the hydraulic oil density. We assume $0 < x_i < p_s$, i=3, 4, and $|x_1| < V_0/S$. Besides the four state variables defining the valve-actuator-load system, it was considered a first order dynamics of the EHSV. It has to be said that the synthesis of the control variable for EHS is far from being trivial, firstly due to strongly nonlinear hydraulic dynamics, but also due to the switching character of valve ports opening and, herein, due also to considering the delay on input (control) variable; the latter two issues were the subject of special attention in this article.

Let's now prepare the system for applying the machinery of Lyapunov equilibrium stability. Accordingly, first the Jacobian matrices of the two component subsystems are calculated Eqs. (11) and (12). Let x_0, positive or negative, be an equilibrium point for the state variable x_1, with the fulfilment of the condition $|x_0| < V_0/S$ regarding the piston stroke. In fact, each subsystem will have its equilibrium point, compatible with the direction of the piston rod movement, from left to right or vice versa, with the sign of state variable x_5 and with the conditions to ensure the positivity of the quantities under radicals. For example, for subsystem 1, defined by the condition $x_5 > 0$, an initial position of the load is selected, $\hat{x}_{1,1} = x_{0,1}$, the load velocity will be naturally $\hat{x}_{2,1} = 0$ and, with a choice of pressures pair $\hat{x}_{3,1} = p_s/2 + kx_{0,1}/(2S)$, $\hat{x}_{4,1} = p_s/2 - kx_{0,1}/(2S)$ to ensure zero velocity, the fifth state equilibrium $\hat{x}_{5,1}$ will be a solution of the equation $Cx_5\sqrt{(p_s - kx_{0,1}/S)/2} - k_l kx_{0,1}/S = 0$. The calculations are similar for subsystem 2 defined by the condition $x_5 < 0$, with the same choice of the equilibrium $\hat{x}_{1,2} = x_{0,2}$, $\hat{x}_{2,2} = 0$ and so on. Equilibrium conditions are added to control variables: $k_{SV} u_{i,0} = x_{5,i}$, $i = 1, 2$. Translate these equilibria into zero by a change of variables

$$y_{1,i} = x_{1,i} - \hat{x}_{1,i},\ y_{2,i} = x_{2,i},\ y_{3,i} = x_{3,i} - \hat{x}_{3,i},$$
$$y_{4,i} = x_{4,i} - \hat{x}_{4,i},\ y_{5,i} = x_{5,i} - \hat{x}_{5,i}, U_i = u_i - u_{i,0}. \tag{13}$$

The systems (11) and (12) is transformed into a system with zero equilibria in the new coordinates system $\boldsymbol{y}$ (but with keeping for simplicity the notation $\boldsymbol{x}$). Let $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ be the Jacobian matrices calculated in zero in the cases $x_5 > 0$, $x_5 < 0$, respectively,*:*

$$\boldsymbol{A}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{BS}{V_0+S\hat{x}_{1,1}} & -\frac{B}{V_0+S\hat{x}_{1,1}}\left(\frac{C\hat{x}_{5,1}}{2\sqrt{p_s-\hat{x}_{3,1}}}+2k_l\right) & 0 & \frac{BC\sqrt{p_s-\hat{x}_{3,1}}}{V_0+S\hat{x}_{1,1}} \\ 0 & \frac{BS}{V_0-S\hat{x}_{1,1}} & 0 & -\frac{B}{V_0-S\hat{x}_{1,1}}\left(\frac{C\hat{x}_{5,1}}{2\sqrt{\hat{x}_{4,1}}}+2k_l\right) & -\frac{BC\sqrt{\hat{x}_{4,1}}}{V_0-S\hat{x}_{1,1}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix} \tag{14}$$

$$\boldsymbol{A}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{BS}{V_0+S\hat{x}_{1,2}} & \frac{B}{V_0+S\hat{x}_{1,2}}\left(\frac{C\hat{x}_{5,2}}{2\sqrt{\hat{x}_{3,2}}}-2k_l\right) & 0 & \frac{BC\sqrt{\hat{x}_{3,2}}}{V_0+S\hat{x}_{1,2}} \\ 0 & \frac{BS}{V_0-S\hat{x}_{1,2}} & 0 & \frac{B}{V_0-S\hat{x}_{1,2}}\left(\frac{C\hat{x}_{5,2}}{2\sqrt{p_s-\hat{x}_{4,2}}}-2k_l\right) & -\frac{BC\sqrt{p_s-\hat{x}_{4,2}}}{V_0-S\hat{x}_{1,2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix} \tag{15}$$

The matrix of control influence $\boldsymbol{B}_c$ is a column vector with the first four elements zero and with the fifth element equal with k_{SV}/τ_{SV}.

There are two objectives of numerical simulation in this Section: (a) the synthesis of the LQR control by the predictive feedback method (see Proposition 2), which is a necessary step to move to the next point (b) the evaluation of the parameter configurations of the EHS systems (11) and (12) (with $\boldsymbol{A}_i$ given by Eqs. (14) and (15)), from the perspective of fulfilling the sufficient stability conditions of the equilibrium (see Theorem 3). Consider the following design data, representing an EHS integrated in the aileron control chain of the jet fighter IAR99 [31,46,47,74]: $m = 30\,\text{kg}$, $f = 3000$ Ns/m, $k = 300\,\text{N/m}$, $\text{S} = 10^{-3}$ m^2, $c_d = 0.63$, $V_0 = 3 \times 10^{-5}$ m^3, $p_s = 210\,\text{N/m}^2$, $B = 13{,}000\,\text{N/m}^2$, $\rho = 850\,\text{kg/m}^3$, $k_{SV} = 2 \times 10^{-4}\,\text{m/V}$ (meaning a maximal opening length of rectangular valve port $x_{5\,\max} = 2$ mm at maximal valve input voltage $u_{\max} = 10$ V, and an equivalent valve port width $w = 0.85\,\text{mm}$), $k_l = 0.04 \times 10^{-11}\text{m}^5/(\text{Ns})$ and τ_{SV} $= 7.62 \times 10^{-3}$ s. The pairs $(\boldsymbol{A}_i, \boldsymbol{B}_c)$, $i = 1, 2$, are not completely controllable, but are stabilizable, including in $x_0 = 0$ (the most vulnerable equilibrium point of a EHS [54]) as indicated the subroutines in the Matlab&Simulink package. In particular, the two matrices $\boldsymbol{A}_i$ Eqs. (14) and (15) are Hurwitz matrices. For the case $x_5 > 0$, choosing $\hat{x}_{1,1} = 5 \times 10^{-3}$m, the following equilibrium vector point is obtained: $\hat{x}_{1,1} = 5 \times 10^{-3}$m, $\hat{x}_{2,1} = 0$ m/s, $\hat{x}_{3,1} = 112.5 \times 10^5$ N/m2, $\hat{x}_{4,1} = 97.5 \times 10^5$ N/m2, $\hat{x}_{5,1} = 0.0018 \times 10^{-3}$m, with $u_1 = 0.0925$. The corresponding eigenvalues of matrix$\boldsymbol{A}_1$ Eq. (14) are: $\lambda_{1,2} = -97.4 \pm 1726.5i$, $\lambda_3 = -0.3$, $\lambda_4 = -90$, and $\lambda_5 = -131.2$. The control law is obtained as example by a simple LQR synthesis [65]. The LQG control synthesis concerns the pairs $(\boldsymbol{A}_i, \boldsymbol{B}_c)$, $i = 1, 2$. Thus, for the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_c u(t)$

with a cost functional defined as $J=\int_0^\infty (\boldsymbol{x}^T(t)\boldsymbol{Q_J}\boldsymbol{x}(t)+R_J u^2(t))dt$ the feedback control law that minimizes the value of the cost is $u(t)=-\boldsymbol{K}_1\boldsymbol{x}(t)$ where $\boldsymbol{K}_1$ is given by $\boldsymbol{K}_1=R_J^{-1}\boldsymbol{B}_c\boldsymbol{P}$ and $\boldsymbol{P}$ is found by solving the continuous time algebraic Riccati equation $\boldsymbol{A}_1^T\boldsymbol{P}+\boldsymbol{P}A-\boldsymbol{P}\boldsymbol{B}_cR^{-1}\boldsymbol{B}_c^T\boldsymbol{P}+\boldsymbol{Q}_J=0$. Taking the weighting matrices $\boldsymbol{Q}_J$, as zero matrix excepting $\boldsymbol{Q}_J$ (1,1) = 1 and $R_J=0.0025$, we obtain $\boldsymbol{K}_1=[6.1005\ 0.0002\ 0.0008\ -\ 0.0006\ 3.6293]$ as feedback gain. In closed loop, the eigenvalues of matrix $\boldsymbol{A}_1$ are: $\lambda_{1,2}=-97.4\pm 1726.5i$, $\lambda_3=-10.2,\ \ \lambda_4=-90,\ \ \lambda_5=-130.8$. An analogous procedure is performed for $i=2$.

The key physical parameters in providing sufficient stability conditions $\psi_i(\|\boldsymbol{x}(t)\|)\geq 0$ are described in Theorem 2, x_{0i}[cm], k_l[cm^5/(daNs)], h[s], and are presented in Table 1, along with the contextual mathematical parameters M_i, $\|\boldsymbol{x}(t)\|$, $\|\boldsymbol{R}_i(t)\|$, N_i, $\lambda_{\min}(\boldsymbol{Q}_i)$, $\lambda_{\max}(\boldsymbol{P}_i)$, ω_i. Table data are associated with system component 1. For component 2, close values are obtained, so they are no longer featured for space-saving reasons. A minimal control was used, choosing $R_J=13000$. It should also be noted that in the simulations the unconventional units system (daN, cm, s), suitable for the EHS model, was used.

A few clarifications are helpful. The sizes of matrices $\boldsymbol{Q}_i$ once chosen are correlated by the sizes of matrices $\boldsymbol{P}_i$ by matrix Lyapunov equation. Unfortunately, the two values are always close to each other, making it difficult, even impossible, to ensure a positive expression $\lambda_{\min}(\boldsymbol{Q}_i)-2[\omega_i+\lambda_{\max}(\boldsymbol{P}_i)(M_i\|\boldsymbol{x}\|+N_i)]$. A thorough study [75] reviews dozens of inequalities on the size of matrices $\boldsymbol{Q}_i$ and$\boldsymbol{P}_i$. Such an inequality is the next: $\lambda_{\min}(\boldsymbol{Q}_i)\leq 2\sigma_{\max}(\boldsymbol{A}_i)\lambda_{\max}(\boldsymbol{P}_i)$. Consequently, the size of matrices $\boldsymbol{A}_i$ cannot be significantly controlled in reasonable limits, even if an adimensionalization of systems (14) and (15) is made. The values ω_i are clamped in the relationships $\|\boldsymbol{P}_i\boldsymbol{A}_{di}\|\leq\omega_i<\lambda_{\min}(\boldsymbol{Q}_i)/2,\ i=1,2$, in which the delay h is involved. The values N_i are essentially influenced, in a convenient way, by choosing small enough gains $\boldsymbol{K}_i$. In this situation, the option that remains is to choose rather small disturbances x_{0i} to make the product $M_i\|\boldsymbol{x}\|$ small enough. For remainders of the Taylor series, the exact determination of the upper limits is possible only in the case of simple functions, of a didactic nature. In the case of the nonlinear functions in the third and fourth equations in each of relations (11) and (12), such a calculation of $\boldsymbol{F}_i[\boldsymbol{x}(t)]$ in this paper would be tedious. Instead, numerical assessments have been done, based on numerical simulation of the systems (11) and (12). Several configurations of perturbations $\hat{x}_{j,1},\ j=1,\ldots,5$ have been tested to find those who meet Assumption 2 by fulfilling the conditions described in Theorem 2. It is evident from Table 1 that these conditions, which are sufficient but not necessary for the stability claimed by Theorem 3, are extremely conservative, since they do not allow in the context than values of perturbations very close to the zero solution. The numerical evaluation during the simulation process of the remainders $\|\boldsymbol{F}_i(\boldsymbol{x}(t))\|=\|\dot{\boldsymbol{x}}(t)-\boldsymbol{A}_i\boldsymbol{x}(t)-\boldsymbol{A}_{di}\boldsymbol{x}(t-h)-\boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^0 e^{-\boldsymbol{A}_is}\boldsymbol{B}_cu_i(t+s-h)ds\|$ allowed an intermediate calculation of a global upper bound $M\|\boldsymbol{x}(t)\|^2$, as well as of the two factors $\|\boldsymbol{x}(t)\|, M_i$ of the product, as shown in Table 1 and Fig. 2.

We add that the numerical simulation of system (8) assumed their discretization, starting with the linear system with delay on the control $\dot{\boldsymbol{x}}=\boldsymbol{A}_i\boldsymbol{x}(t)+\ \boldsymbol{B}_c\,u(t-h),\boldsymbol{x}_i(0)=\boldsymbol{x}_{0,i}$, for which the prediction $\boldsymbol{x}(t+h)$ is expressed by relation (6); for the equation with finite differences $\boldsymbol{x}(n+1)=\boldsymbol{A}_{iD}\boldsymbol{x}(n)+\ \boldsymbol{B}_{cD}\,u(n),\boldsymbol{x}_i(0)=x_{0,i}$ the prediction with k steps is expressed through relation $\boldsymbol{x}(n+k)=\boldsymbol{A}_{iD}^k\,x(n)+\sum_{j=n-k}^{n-1}\boldsymbol{A}_{iD}^{\,n-1-j}\,\boldsymbol{B}_{cD}\,u_i(j)$, $\boldsymbol{A}_{iD}:=e^{\boldsymbol{A}_iT}$; $\ \boldsymbol{B}_{cD}:=\int_0^T e^{\boldsymbol{A}\sigma}d\sigma\,\boldsymbol{B}_c=\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_iT}-I)\boldsymbol{B}_c$, with $\boldsymbol{A}_i$ invertible. T is sampling time, $h=kT,\ u_n:=u(nT)$, for $nT\leq t\leq(n+1)T,\ \ T>0,\ \ n=0,\ 1,\ 2,\ \ldots$ Discretization is ex-

Table 1
Configurations of key physical and mathematical parameters for stability evaluation, see Theorem 2.

	x_{01}	k_l	h	M_1	$\|x(t)\|$	$\|R_1\|$	N_1	$\lambda_{\min}(Q_1)$	$\lambda_{\max}(P_1)$	ω_1	$\Psi_1(\|x\|)$
1	0.01	0.1	0.1	61.9×10^{-3}	0.212	0.013	1.3×10^{-8}	80	9.2×10^{4}	39.301	-1.1×10^{3}
2	0.01	0.1	0.1	$63.\ \times\ 10^{-3}$	0.212	0.013	1.3×10^{-8}	10	1.15×10^{4}	4.913	-13.657
3	0.001	0.1	0.1	6.2×10^{-3}	0.021	1.3×10^{-4}	1.3×10^{-8}	80	9.22×10^{4}	39.421	-0.010
4	0.001	0.5	0.1	116×10^{-3}	0.023	2.5×10^{-3}	$5.\ \mathrm{x}10^{-10}$	80	2.03×10^{4}	1.790	-0.018
5	0.001	0.05	0.1	15.7×10^{-3}	0.021	3.3×10^{-4}	5.2×10^{-8}	80	1.78×10^{4}	144.17	$-\ 0.155$
7	0.0005	0.1	0.1	3.1×10^{-3}	0.011	3.3×10^{-5}	1.3×10^{-8}	80	9.22×10^{4}	39.37	-5.4×10^{-4}
8	0.0003	0.1	0.1	1.86×10^{-3}	6.4×10^{-3}	1.8×10^{-5}	1.3×10^{-8}	80	9.2×10^{4}	39.43	-4.2×10^{-5}
9	0.0001	0.1	0.1	6.2×10^{-4}	2.1×10^{-3}	1.3×10^{-6}	1.3×10^{-8}	80	9.2×10^{4}	39.43	$\mathbf{4.\ \times\ 10^{-6}}$
10	0.0001	0.1	0.1	6.2×10^{-4}	2.1×10^{-3}	1.3×10^{-6}	1.3×10^{-8}	2000	2.3×10^{6}	985.8	$\mathbf{1.\ \times\ 10^{-4}}$
11	0.0001	0.1	0.1	6.2×10^{-4}	2.1×10^{-3}	1.3×10^{-6}	1.3×10^{-8}	13,000	1.5×10^{7}	6.49×10^{3}	$\mathbf{6.5\ x\ 10^{-4}}$
12	0.0001	0.1	0.1	6.2×10^{-4}	2.1×10^{-3}	1.3×10^{-6}	1.3×10^{-8}	100,000	1.10×10^{8}	4.93×10^{4}	**0.0050**

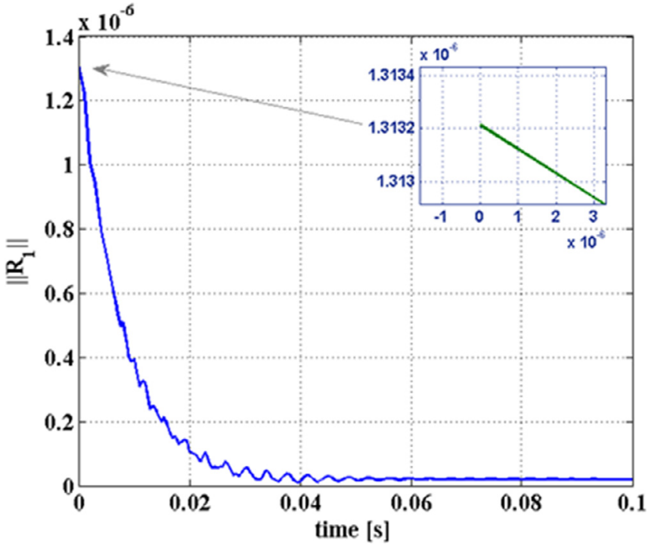

Fig. 2. Evolution of the norm $\|\mathrm{R_1}\|$ in the case $\mathbf{x}_{01} = 0.0001\,\mathrm{cm}$.

tended in this way to the entire nonlinear system. The evolution of the state x_5 sign is monitored to connect in simulation one or the other component of the switching system.

Synoptic graphs in Figs. 3–5 give another perspective on the stability of system (8), compared to the drastic conditions in Table 1. Table 1 shows the result of a mathematical demonstration, while Figs. 3–5 present the result of a numerical experiment.

As outlined above, the problem is that those asymptotic stability conditions in Theorem 2 are extremely conservative, which is almost a rule with the sufficient conditions, generally speaking. Therefore, in Fig. 3 it can be seen that the mathematical model (8) of the EHS, without delay and with a LQR control law, denotes an accentuated stability, practically indifferent to the magnitude of the perturbation (in the graphs in Figs. 3–5 the perturbations are substantial, $\hat{x}_{1,1} = 0.5\mathrm{cm}$ or $\hat{x}_{1,2} = -0.5\mathrm{cm}$), and the distinction between linear and nonlinear is insignificant. Fig. 4 highlights what can happen in the case of actuator delay, without predictive control. Moreover, a nearly 0.1 s threshold value of the delay is identified. For $h = 0.096\,\mathrm{s}$, we have a pole in discrete time at the stability limit, $z = 0.99995$. For h^* $= 0.1\,\mathrm{s}$, we have an unstable pole $z = 1.0002346$. Finally, the synthesis methodology of the control law, as well as the numerical simulation procedure, are validated by the graphs in Fig. 5. It is noteworthy that, if we overlook the portion of the graphs corresponding to the 0.1 s delay, the graphs in Figs. 3 and 5 are very close to each other. It is as if the system without delay is practically the same as the delayed system compensated by the control law of predictive feedback. This happen despite of the control law concept, essentially different from [56].

A last point of interest for numerical applications could be the asymptotic stability assessment of the homogeneous linear state delay system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{di}\boldsymbol{x}(t-h), \boldsymbol{A}_{di} :$ $= \boldsymbol{B}_c\boldsymbol{K}_i e^{\boldsymbol{A}_i h}$ $(*)$. Stability of such an equation is ensured if all zeros of the transcendental characteristic equation $\det(s\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}_d e^{-\boldsymbol{I}sh}) = 0$ fulfil the $\mathrm{Re}s > 0$ inequality (see, e.g., [2]), in which $\boldsymbol{I}$ is the identity matrix. To avoid tedious calculations for analysing the solutions of this transcendent equation, it is appropriate to use a result from [76] that says the asymptotic stability of the equation $(*)$ occurs if there exists of least one solution for the nonlinear

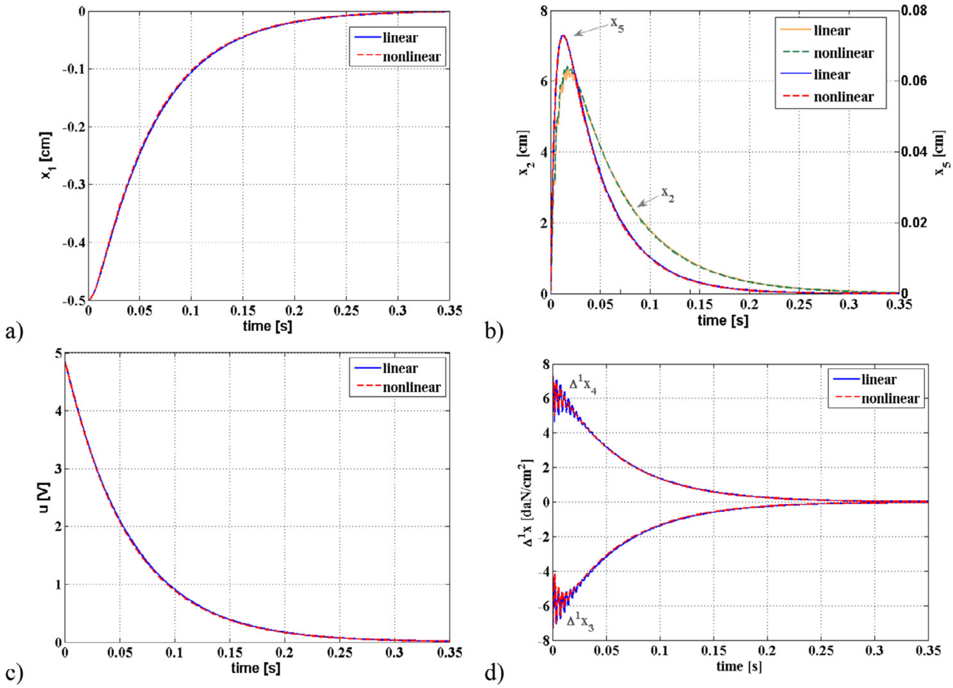

Fig. 3. History of system variables, the case of the EHS without delay.

Table 2
The existence of the solution of the transcendental equation $e^{(\boldsymbol{A}_1+\boldsymbol{P}_1(0))h}\boldsymbol{P}_1(0)=\boldsymbol{A}_{d1}$.

#	*h*	*solution checking*	*conclusion*
1	0.005	1.73×10^{-10}	there is a solution
2	0.01	2.18×10^{-9}	there is a solution
3	0.015	1.99×10^{-10}	there is a solution
4	0.02	7.52	there is no solution
5	0.09	2.74×10^{4}	there is no solution

algebraic matrix equation $e^{(\boldsymbol{A}_1+\boldsymbol{P}_1(0))h}\boldsymbol{P}_1(0)=\boldsymbol{A}_{d1}$. A simple numerical investigation of this equation is done by using Matlab fsolve subroutine and is summarized in Table 2. This shows the method efficiency as compared to more laborious approaches in [77,78] in which is used the Lambert function.

The result in the Table says something about the risk of loss of the stability for the system $\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)-\boldsymbol{A}_{di}\boldsymbol{x}(t-h)$. Unfortunately, it is all about sufficient stability conditions, which can also be very conservative. However, we can take advantage of the results from Table 2 in the sense that the stability threshold can climb up to $h=0.1$ s, if we take into account that in the analysed equation (*) lacks the compensating term $\boldsymbol{B}_c\boldsymbol{K}_i\int_{-h}^{0}e^{-\boldsymbol{A}_is}\boldsymbol{B}_cu_i(t+s-h)ds$.

5. Concluding remarks

The novelty of the present study consists in addressing and solving a problem of equilibrium stability for a nonlinear structural switching system with actuator delay. A first re-

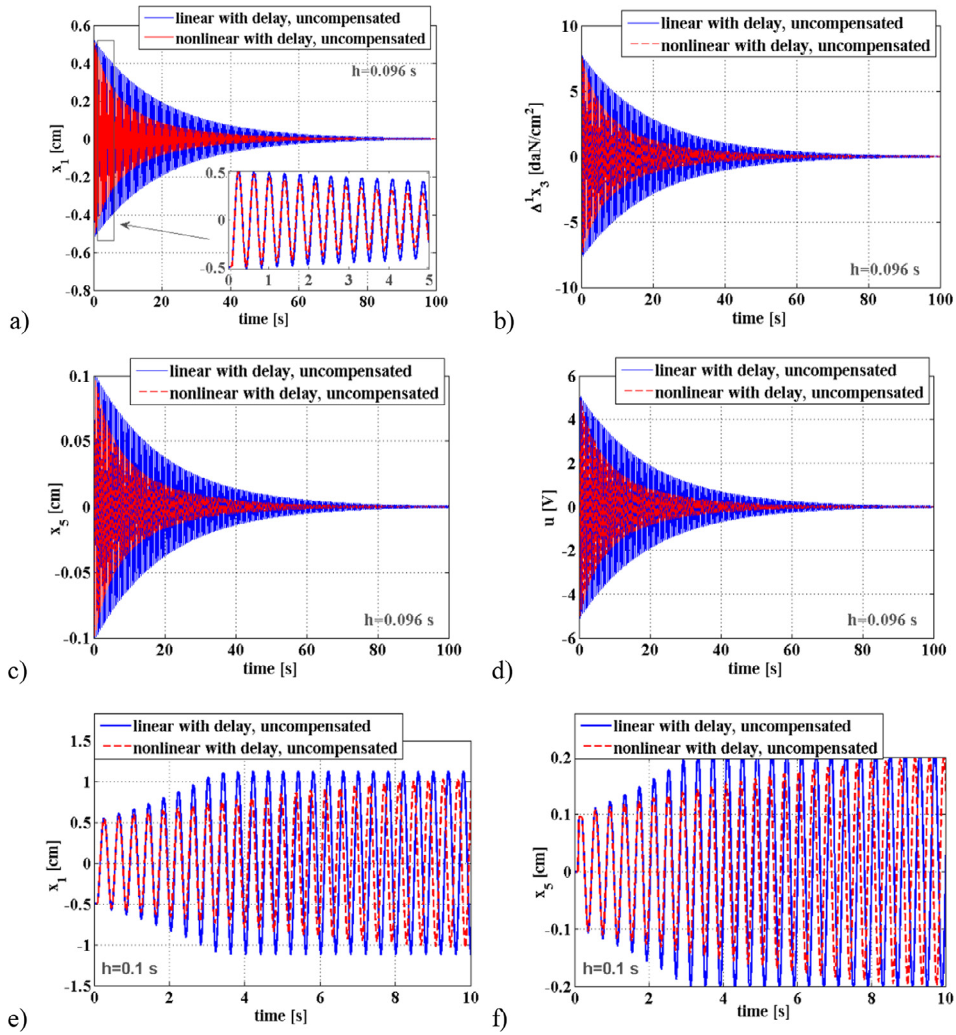

Fig. 4. History of variables for the switching system with simple LQR feedback.

sult of the paper is described by Theorems 2 and 3 which give sufficient conditions for asymptotic stability of this equilibrium. Unfortunately, the stability conditions in Theorem 2 have proved to be extremely conservative, which is a rule with conditions that are only sufficient, but not necessary. A second result refers to numerical applications on such a consecrated system, the mathematical model of the EHS. In this way, the present paper continues some works published in the field of hydraulic servomechanisms analysis and synthesis [28–32,44,46–50,74,79–82]. Note that it is for the first time, to the best of our knowledge, when the study of the actuator delay is corroborated with that of structural switching in the EHS mathematical model. This mathematical models (11) and (12) has on the one hand an improvement with respect to the model in [31] and on the other hand introduces the delay on the control variable. The improvement, otherwise realistic [55], refers to taking

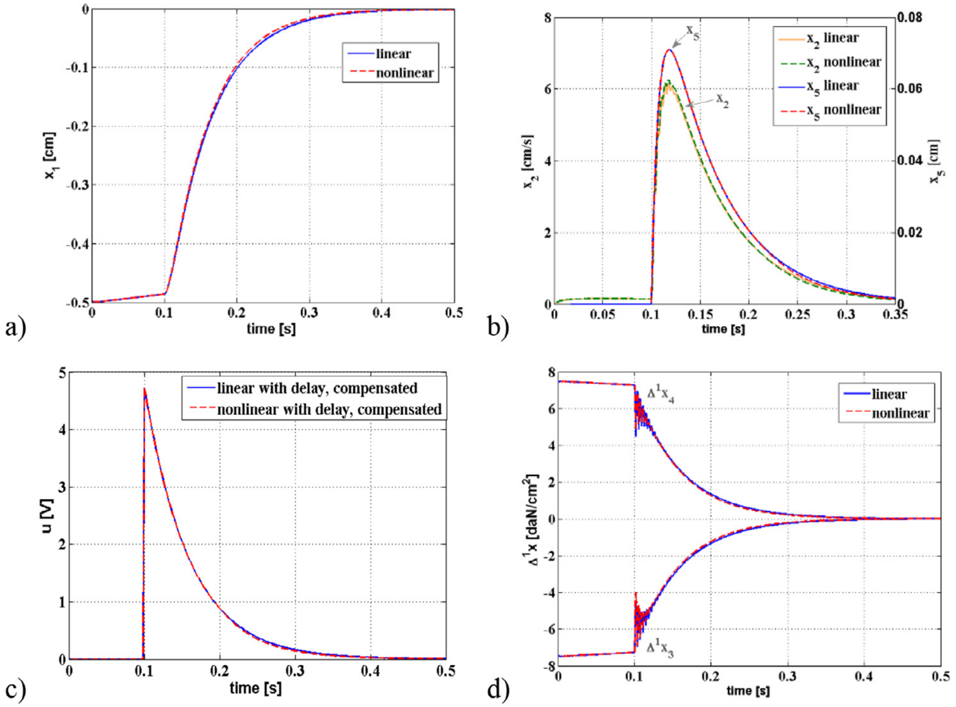

Fig. 5. History of variables for switching system with predictive feedback control.

the advantage of a certain “flexibility” of mathematical models [47,48,81] by introducing the leakages $k_l \neq 0$. Otherwise, matrices $\boldsymbol{A}_i$ would each have two null eigenvalues and, in this context, in [31] it was necessary to resort to the apparatus Lyapunov–Malkin of critical cases of stability. A next study could address all three aspects simultaneously: critical case of stability, structural switching, and actuator delay. Also, the question of conservativeness reducing remains a target in a next approach to the stability. The main result of numerical simulations is the maximum allowable delay value $h^* = 0.1$ s, beyond which stability is lost. In the literature of the field we have no other term of comparison, than recent work [82], in which the same value is confirmed by specific simulations performed by model discretization.

Noting the competition with other types of actuators – electromechanical actuators, hydrostatic actuators, piezo actuators (see the so-called “green aircraft” and details in [83]) –, we believe that the EHSs will still be for many years the right choice in many application domains.

The EHS is in fact a position tracking system, but the EHS as a stabilizing system can be viewed as a special case of the tracking system [47]. An eloquent example of practical interest for EHS as a stabilizing system is the well -known problem of altitude-hold autopilot synthesis, involving an EHS, where the target is the maintenance of the desired altitude of the aircraft, thus allowing the pilot to perform other more important tasks. An earlier approach of the closely related EHS equilibria stability problem has been proposed in the framework of the absolute stability theory, as stated by Aizerman, Lurie, Lefschetz and Popov [84].

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