Intermittency Analysis and Spatial Dependence of Magnetic Field Disturbances in the Fast Solar Wind

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Yang and Tam [JASTP, 2010], a paper that is forgotten …

by both authors!

because of:

1. Double ROMA [Tam et al., 2010]
2. New challenges in the career
Outline

• Data Selection
  – Helios 1 and 2 spacecraft

• Intermittency Analyses of Magnetic Field
  – mean field vs. fluctuations
  – magnitude vs. components

• Summary

• Spatial Dependence of Magnetic Field Fluctuations

• Summary
Data Selection

- Heliocentric distances: 0.29-1.0 AU
Data Selection (cont.)

• Identification of individual fast solar wind events

1. intervals with hourly averaged solar wind speed \( V \geq 550 \text{ km/s} \) basically throughout; isolated gaps due to slow speeds or missing data of less than one day accepted (fast SW)

2. at least 7200 data points of 6-second averaged magnetic field available in the interval (sufficient statistics)

3. one event splitting into two potential events when the change in \( V \) between two adjacent points (1 hr apart) is greater than 30 km/s and \( V \) changes by more than 60 km/s over two data points (2 hrs apart) (to reduce the chance of one event splitting over different coherent structures)
Three selection criteria:

1. $|V_i| \geq 550$ km/s

2. $N_B > 7200$ points

3. $|V_{i-V_{i-1}}| > 30$ km/s & $|V_{i+1-V_{i-1}}| > 60$ km/s

- 39 events in all, details in *Yang and Tam [JASTP, 2010]*
Intermittency Analyses of Magnetic Field

• For each event, find the mean magnetic field, \( \langle B \rangle \), by taking the average of all the data points for each of the three magnetic field components.

• Determine the perturbed magnetic field as the fluctuations about the mean field, \( B_f = B - \langle B \rangle \).

• Seven field quantities to be analyzed for each event:

\[
\begin{align*}
|B| & : \text{magnitude of the total measured magnetic field} \\
|B_f| & : \text{magnitude of the perturbed magnetic field} \\
|B_{f,\perp}| & : \text{magnitude of perturbed magnetic field components transverse to } \langle B \rangle \\
B_{f,\parallel} & : \text{parallel component of the perturbed magnetic field} \\
B^2 & : \text{magnetic energy density of the total measured field} \\
B_f^2 & : \text{magnetic energy density of the perturbed field} \\
B_{f,\perp}^2 & : \text{magnetic energy density of the perpendicular components of the perturbed field}
\end{align*}
\]
**Intermittency Analyses**

* PDF (Probability Distribution Function):
  - For a time series of a field quantity \( X(t) \), generate PDF \( P(\delta X, \tau) \) for different time scales \( \tau \), where \( \delta X \equiv X(t + \tau) - X(t) \)
  - To compare PDF of different \( \tau \), useful to consider the normalized PDF (variance = 1):
    \[
    P^*_\delta X/\sigma, \tau = \sigma P(\delta X, \tau)
    \]
    where \( \sigma(\tau) = \sqrt{\langle (\delta X)^2 \rangle} \)
• Fitting $P_*$ with a Castaing distribution [Castaing et al., 1990]:

$$
\Pi_\lambda (\xi) = \frac{1}{2\pi \lambda} \int_0^\infty \frac{d\alpha}{\alpha^2} \exp \left( -\frac{\xi^2}{2\alpha^2} \right) \exp \left( -\frac{\ln^2 (\alpha/\alpha_0)}{2\lambda^2} \right)
$$

letting $\ln \alpha_0 = -\lambda^2$ such that the variance of $\Pi_\lambda (\xi)$ is 1, same as that of $P_*$

• Find the optimal value of $\lambda$ by least square fitting, where $\lambda > 0$ and characterizes the degree of intermittency

$\lambda = 0$: Gaussian

Degree of intermittency increases with $\lambda$

![Graph showing the distribution $\Pi_\lambda (\xi)$ for different values of $\lambda$.](image)
**Flatness:**

\[
F\{P_*(\delta X / \sigma, \tau)\} = \frac{< (\delta X / \sigma)^4 >}{< (\delta X / \sigma)^2 >^2} = \frac{\int (\delta X / \sigma)^4 [P_*(\delta X / \sigma, \tau)] d(\delta X / \sigma)}{\left(\int (\delta X / \sigma)^2 [P_*(\delta X / \sigma, \tau)] d(\delta X / \sigma)\right)^2}
\]

\[
= \frac{\int (\delta X / \sigma)^4 [\sigma P(\delta X, \tau)] d(\delta X / \sigma)}{\left(\int (\delta X / \sigma)^2 [\sigma P(\delta X, \tau)] d(\delta X / \sigma)\right)^2} = \frac{\int (\delta X)^4 P(\delta X, \tau) d(\delta X)}{\left(\int (\delta X)^2 P(\delta X, \tau) d(\delta X)\right)^2} = \frac{< (\delta X)^4 >}{< (\delta X)^2 >^2}
\]

\[
= F\{P(\delta X, \tau)\}
\]

- The Flatness $F$ increases with the degree of intermittency of the distribution.
- For Gaussian distributions, $F = 3$; for Castaing distributions, $F = 3 \exp(4\lambda)$
• Present study [Yang and Tam, 2010]
  – 39 events throughout the Helios mission period, heliocentric distances 0.29 – 1 AU, 10 different \( \tau \) ‘s (6, 12, 24, …, 3072 sec)

• Previous intermittency analyses of fast solar wind magnetic fields based on PDF or Flatness:

  • Sorriso-Valvo [1999]
  • a 4-month period of Helios 2 data, heliocentric distance changing from 1 AU to 0.29 AU
  • PDF on \( \delta|B| \)
  • intermittency decreases with larger \( \tau \), PDF approaching Gaussian
• **Bruno et al. [2003]**
  - three events of the same corotating stream at three different heliocentric distances
  - Flatness on $\delta |B|$ (compressive fluctuations) and $\delta B$ (directional fluctuations, equivalent to $\delta |B_f|$ in this study)
  - compressive fluctuations more intermittent than directional fluctuations
  - intermittency increases with heliocentric distance
Normalized PDF Fitting

\[ \delta B \]

\[ \tau = 6 \text{ s} \quad \tau = 12 \text{ s} \quad \tau = 24 \text{ s} \quad \tau = 48 \text{ s} \quad \tau = 96 \text{ s} \quad \tau = 192 \text{ s} \quad \tau = 384 \text{ s} \quad \tau = 768 \text{ s} \]

\[ \delta [B] / \sigma \]

\[ \delta B \]

\[ \delta fB \]

\[ \delta \perp fB \]

\[ \delta fB \]

\[ \sigma = 1.53 \quad F = 51.50 \quad \sigma = 1.64 \quad F = 45.02 \quad \sigma = 1.74 \quad F = 42.40 \quad \sigma = 1.84 \quad F = 36.20 \quad \sigma = 1.94 \quad F = 30.12 \quad \sigma = 2.05 \quad F = 25.70 \quad \sigma = 2.18 \quad F = 19.57 \quad \sigma = 2.35 \quad F = 15.29 \]

\[ \chi^2 = 2.32 \quad \lambda = 1.10 \quad \chi^2 = 2.02 \quad \lambda = 1.01 \quad \chi^2 = 1.98 \quad \lambda = 1.03 \quad \chi^2 = 1.92 \quad \lambda = 0.99 \quad \chi^2 = 2.10 \quad \lambda = 0.94 \quad \chi^2 = 1.75 \quad \lambda = 0.86 \quad \chi^2 = 1.43 \quad \lambda = 0.84 \quad \chi^2 = 1.87 \quad \lambda = 0.79 \]

\[ \delta B_f \]

\[ \sigma = 4.90 \quad F = 10.36 \quad \sigma = 6.66 \quad F = 7.30 \quad \sigma = 8.43 \quad F = 5.50 \quad \sigma = 10.11 \quad F = 4.44 \quad \sigma = 11.55 \quad F = 3.86 \quad \sigma = 12.64 \quad F = 3.55 \quad \sigma = 13.34 \quad F = 3.39 \quad \sigma = 13.63 \quad F = 3.26 \]

\[ \chi^2 = 1.32 \quad \lambda = 0.60 \quad \chi^2 = 1.49 \quad \lambda = 0.51 \quad \chi^2 = 1.95 \quad \lambda = 0.57 \quad \chi^2 = 1.99 \quad \lambda = 0.25 \quad \chi^2 = 2.57 \quad \lambda = 0.21 \quad \chi^2 = 1.58 \quad \lambda = 0.15 \quad \chi^2 = 1.45 \quad \lambda = 0.17 \quad \chi^2 = 1.15 \quad \lambda = 0.13 \]

\[ \delta B_{f,1} \]

\[ \sigma = 4.88 \quad F = 7.91 \quad \sigma = 6.62 \quad F = 5.77 \quad \sigma = 8.36 \quad F = 4.49 \quad \sigma = 10.04 \quad F = 3.75 \quad \sigma = 11.44 \quad F = 3.29 \quad \sigma = 12.48 \quad F = 3.06 \quad \sigma = 13.20 \quad F = 2.89 \quad \sigma = 13.60 \quad F = 2.60 \]

\[ \chi^2 = 1.63 \quad \lambda = 0.52 \quad \chi^2 = 1.52 \quad \lambda = 0.39 \quad \chi^2 = 2.42 \quad \lambda = 0.33 \quad \chi^2 = 2.89 \quad \lambda = 0.20 \quad \chi^2 = 2.99 \quad \lambda = 0.14 \quad \chi^2 = 3.11 \quad \lambda = 0.14 \quad \chi^2 = 3.60 \quad \lambda = 0.11 \quad \chi^2 = 2.69 \quad \lambda = 0.01 \]

\[ \delta B_{f,2} \]

\[ \sigma = 4.38 \quad F = 14.51 \quad \sigma = 6.16 \quad F = 11.19 \quad \sigma = 7.89 \quad F = 8.72 \quad \sigma = 9.51 \quad F = 6.99 \quad \sigma = 10.92 \quad F = 6.24 \quad \sigma = 11.94 \quad F = 5.57 \quad \sigma = 12.65 \quad F = 5.27 \quad \sigma = 13.01 \quad F = 5.15 \]

\[ \chi^2 = 1.72 \quad \lambda = 0.71 \quad \chi^2 = 1.50 \quad \lambda = 0.62 \quad \chi^2 = 1.33 \quad \lambda = 0.54 \quad \chi^2 = 2.05 \quad \lambda = 0.45 \quad \chi^2 = 1.89 \quad \lambda = 0.40 \quad \chi^2 = 2.00 \quad \lambda = 0.40 \quad \chi^2 = 2.36 \quad \lambda = 0.35 \quad \chi^2 = 1.82 \quad \lambda = 0.37 \]

less intermittent
• All 39 events feature the trends below
• For the same time scale, \( \delta|B| \) and \( \delta(B^2) \) are more intermittent than the other quantities, consistent with the results by Bruno et al. [2003]
• For all the magnetic field quantities, both \( F \) and \( \lambda \) show decreasing trends as \( \tau \) increases, consistent with results by Sorriso-Valvo [1999]
• Quantities associated with the perturbed magnetic field feature more apparent changes in the shape of the normalized PDF as \( \tau \) increases, \( P_* \) becoming close to a Gaussian distribution \((F = 3 \text{ or } \lambda = 0)\) at much smaller \( \tau \)
Variation of Flatness with Distance

Flatter slopes lasting at close scale increases
General Trends between $F$ and distance:

1. Positive slopes
   - Magnetic field turbulence more intermittent at larger heliocentric distances (consistent with the results by Bruno et al. [2003])

2. Flatter slopes as the time scale increases
   - For all the quantities, $F$ increases by a lesser extent with distance as the time scales increases
Variations of Flatness with time scale in different distance ranges

- As the time scale decreases, the increase in $F$ is larger for distances farther away from the Sun
- Magnetic field turbulence more intermittent farther away from the Sun
General Trends between $F$ and distance:

1. Positive slopes
   - Magnetic field turbulence more intermittent at larger heliocentric distances (consistent with the results by *Bruno et al.* [2003])

2. Flatter slopes as the time scale increases
   - For all the quantities, $F$ increases by a lesser extent with distance as the time scales increases
   - Reason: PDF’s generally approach Gaussian distribution with increasing time scales. At large time scales, even events at small distances have $F$ already falling to the order of the Gaussian value of 3; as the time scale becomes smaller, $F$ increases by more at distances farther away from the Sun (turbulence more intermittent at larger distances)
Flatness Comparison among Quantities

Total magnetic field magnitude $|\mathbf{B}|$ has larger $F$ values than the perturbed field quantities, $|\mathbf{B}_f|$, $|\mathbf{B}_{f,\perp}|$, and $B_{f,\parallel}$.
Many occasions when $B_{f,\perp}$ flips sign while $|\delta B_{f,\perp}|$ is small.

Require very large $|\delta B_f|$ with specific combinations of $\delta B_{f,\perp}$ and $\delta B_{f,\perp}$ OR small $|\delta B_{f,\perp}|$ with a considerably large positive $|\delta B_{f,\perp}|$.

$\delta |B_f|$ large, but $\delta |B|$ small

$B_{f,\perp}$ flips sign

$\delta |B|$ large, but $\delta |B_f|$ small
\[ |B| = \sqrt{(\langle B \rangle + B_{f,\perp})^2 + |B_{f,\perp}|^2} \]

large \( \delta|B_{f,\perp}| \) with small \( \delta B_{f,\perp} \) \( \implies \) large \( \delta|B| \) but small \( \delta B_{f,\perp} \)

large \( \delta B_{f,\perp} \) with small \( \delta|B_{f,\perp}| \) \( \implies \) large \( \delta|B| \) but small \( \delta|B_{f,\perp}| \)
Perturbed magnetic field magnitude has $F$ values comparable to (but generally smaller than) those of $|\mathbf{B}_f|$ in most events.

Parallel component of perturbed field $B_{f,\parallel}$ has larger $F$ values than $|\mathbf{B}_{f,\perp}|$ in all events.
A better way to compare intermittency of parallel and perpendicular perturbed fields:

- To consider the perpendicular fluctuations in individual directions rather than the magnitude in the 2-D plane
- Arbitrary choose two orthogonal directions, \( \hat{e}_{\perp 1} \) and \( \hat{e}_{\perp 2} \), in the plane perpendicular to the mean magnetic field, and determine \( B_{f,\perp 1} \) and \( B_{f,\perp 2} \) accordingly
Component-wise, the parallel fluctuations $B_{f,\parallel}$ are still more intermittent than the perpendicular fluctuations.
The dominance in the $F$ values is rather evenly divided between the magnetic energy densities of the perturbed field and its perpendicular component.
Summary: Intermittency Analyses

- Investigation of magnetic field intermittency in the fast solar wind using magnitude of the total measured field as well as quantities associated with the perturbed field.
- Intermittency increases at smaller scales and at distances farther away from the Sun.
- The total magnetic field magnitude and energy density are more intermittent than the perturbed field quantities.
- Among the components, the perturbed field in the direction parallel to the mean magnetic field is more intermittent than those in the perpendicular directions.
- The degree of intermittency of the magnetic energy densities of the perturbed field and its perpendicular components is comparable.
Spatial Dependence of Magnetic Field Fluctuations

Motivation

For any given $\tau$, the rms of $\delta |B_f|$ is approximately proportional to the mean magnetic field.

$$\frac{\sigma(\delta |B_f|)}{\langle |B| \rangle}$$ depends strongly on $\tau$ and weakly on heliocentric distance.

Points from left to right at each level of the mean magnetic field:
$\tau = 6, 12, 24, \ldots, 3072$ s

It should be interesting to examine the relationship between $|B_f|^2$ and $\langle |B| \rangle^2$ at different time scales.
Spatial Dependence of Magnetic Field Fluctuations

Approach

• Use the magnitude of the mean magnetic field $|\langle B \rangle|$ as a proxy for the distance ($|\langle B \rangle|$ larger, distance smaller)

• Consider $|B_f(1/\tau)|^2 = |B_{f,x}(1/\tau)|^2 + |B_{f,y}(1/\tau)|^2 + |B_{f,z}(1/\tau)|^2$, the spectral density associated with different time scales $\tau$, based on the Fourier transform

• Examine how $|B_f(1/\tau)|^2 / |\langle B \rangle|^2$ at different $1/\tau$ varies with $|\langle B \rangle|^2$
$\log(|B_f|^2) \text{ vs. } \log(|\langle B \rangle|^2)$

$\log(|B_f (1/\tau)|^2) = A_0 + A_1 \log(|\langle B \rangle|^2)$

<table>
<thead>
<tr>
<th>$\tau$ (sec)</th>
<th>$A_0$</th>
<th>$A_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$-7.346 \pm 0.106$</td>
<td>$1.022 \pm 0.051$</td>
</tr>
<tr>
<td>24</td>
<td>$-6.903 \pm 0.106$</td>
<td>$1.038 \pm 0.051$</td>
</tr>
<tr>
<td>48</td>
<td>$-6.334 \pm 0.107$</td>
<td>$1.009 \pm 0.051$</td>
</tr>
<tr>
<td>96</td>
<td>$-5.811 \pm 0.100$</td>
<td>$0.987 \pm 0.052$</td>
</tr>
<tr>
<td>192</td>
<td>$-5.380 \pm 0.116$</td>
<td>$1.009 \pm 0.055$</td>
</tr>
<tr>
<td>384</td>
<td>$-4.817 \pm 0.090$</td>
<td>$0.963 \pm 0.043$</td>
</tr>
<tr>
<td>768</td>
<td>$-4.173 \pm 0.098$</td>
<td>$0.857 \pm 0.047$</td>
</tr>
<tr>
<td>1536</td>
<td>$-3.627 \pm 0.103$</td>
<td>$0.775 \pm 0.048$</td>
</tr>
<tr>
<td>3072</td>
<td>$-3.218 \pm 0.114$</td>
<td>$0.756 \pm 0.055$</td>
</tr>
</tbody>
</table>

$|B_f (1/\tau)|^2 = 10^{A_2} (|\langle B \rangle|^2)^{A_1} (1/\tau)^{-A_3}$

$A_3$: power-law index for the spectral density
\[
\log\left(\frac{|B_f(1/\tau)|^2}{|\langle B \rangle|^2}\right) = A_0 + (A_1 - 1) \log(|\langle B \rangle|^2)
\]

\(A_1 > 1: \frac{|B_f(1/\tau)|^2}{|\langle B \rangle|^2}\) decreases with distance

\(A_1 < 1: \frac{|B_f(1/\tau)|^2}{|\langle B \rangle|^2}\) increases with distance

\(|A_1 - 1|\) characterizes the extent of the change

\(\tau = 3072\) s

Spectral density favoring larger time scale (smaller frequency) as the heliocentric distance increases
Could the observed spatial dependence be due to wave/fluctuation-particle interaction?

Almost all of the spectral densities considered in the study are at frequencies below the ion cyclotron frequency.

**Ion cyclotron resonance condition:** \( \omega = \Omega_i + k \cdot v \)

In the spacecraft frame, \( \omega < \Omega_i \), resonance requires \( k \cdot v < 0 \) i.e., \( k \) and \( v \) in opposite directions.
In the spacecraft frame, \( \nu \) is anti-sunward, requiring sunward \( k \) for ion cyclotron resonance.

The Alfvén speed was found to be much smaller than the solar wind speed in all the events, sunward \( k \) in the spacecraft frame is therefore highly improbable.

The observed spatial dependence of the spectral density could not be the results of ion cyclotron resonance, thus mostly likely due to fluctuation-fluctuation (wave-wave) interaction.
As the fluctuations have anti-sunward $k_\parallel$ in the spacecraft frame, they go from small to large heliocentric distances, where the spectral density increasingly favoring large scales, while interacting.

Our observed spatial dependence of the spectral density suggests that the fluctuation-fluctuation interaction redistributes their spectral density among the various scales, favoring the larger scales (from 384 sec to at least 3072 sec; spatial scales: multiply the time scales by the solar wind speed).
Summary: Spatial Dependence of Magnetic Field Fluctuations

• The normalized spectral densities for larger scales increase as the heliocentric distance becomes larger.
• The spatial dependence of the spectral densities does not seem to be due to ion cyclotron resonance, thus most likely due to fluctuation-fluctuation interaction.
• Fluctuation-fluctuation interaction redistribute their spectral density ("energy") in favor of the larger scales considered in this study.
• The results may provide guidelines for the development of theories and simulations for fluctuation-fluctuation interaction in the fast solar wind.