Introduction to

Space Plasma Complexity

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a) $B^2$ along $y=\pi$

b) $\log_{10}$ NP for $B^2$ Wavelets
ALFVÉN WAVES

\[\frac{d\mathbf{B}}{dt} = (\mathbf{B} \cdot \nabla) \mathbf{V}\]

\[\rho \frac{d\mathbf{V}}{dt} = (\mathbf{B} \cdot \nabla) \mathbf{B} + \cdots\]
ALFVÉN RESONANCES

\[ B \cdot \nabla \rightarrow i k \cdot B = 0 \]
SOLAR WIND

CHANG (1998)
BRUNO ET AL. (2001)
ALEXANDROVA 2008
Coarse-Grained Dissipation
And
Magnetic Reconfiguration

WU & CHANG, 2001
COARSE-GRAINED DISSIPATION
PARAMETER SET: \{P\}

DRG TRANSFORMATION:

d\{P\}/dl = R\{P\}

Procedure:
Search for Invariants
LINEAR TRANSFORMATIONS

→ POWER LAW INVARIANTS.

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EXAMPLE:

\[ P \rightarrow e^{\alpha_P} P, \quad E \rightarrow e^{\alpha_E} E \] LEADS TO

\[ P/E^{\alpha_P/\alpha_E} \] AS AN INVARIANT.
HOW DO WE DEFINE \textbf{DIMENSIONS}?
$d = \frac{\log 4}{\log 2} = 2$
SIERPINSKI TRIANGLE (1916)
\[ d = \frac{\log 3}{\log 2} \approx 1.58 \]
FLARE FREQUENCY
(events per count s\(^{-1}\) per day)

PEAK COUNTING RATE (counts s\(^{-1}\))
\[ P(E, L) \]
\[ \frac{P}{E^a} = I \]
\[ E / L^b = J \]
\[ \rightarrow I = F(J) \]

**Finite Size Scaling (FSS)**
a.
COARSE-GRAINED MEASURE: STRUCTURE FUNCTIONS

\[ S_q \equiv \left\langle B(x_i + \Delta) - B(x_i) \right\rangle^q \sim \Delta^\zeta_q , \quad \Delta = k\delta \]
Bruno et al. 2007, Magnetic field strength fluctuations
Fixed Points: $\frac{d\{P_N\}}{d\ell} = 0$
PROBABILITY DISTRIBUTION FUNCTION OF INCREMENTAL FLUCTUATIONS

\[ P(\delta B^2, \Delta) \]

SCALING NEAR DYNAMIC CRITICALITY

\[ P(\delta B^2, \Delta) \Delta^{-s} = P_s(\delta B^2_s) \]

\[ \delta B^2_s = \delta B^2 / \Delta^s \]
Hnat et al., 2002 (Solar Wind)  
Chang et al., 2004
\[ P(\delta B^2, \Delta) \Delta^{-s} = P_s(\delta B^2_s) \]

\[ Y = \delta B^2_s = \delta B^2 / \Delta^s \quad S = S(Y) \]
ROMA
(Rank Ordered Multifractal Analyses)

Chang and Wu, 2008
Corona EUV Emissions – SOHO (Wu et al., 2011)

Wu, Chang, Uritsky; 2011
ROMA analysis of probability distributions for the SOHO EIT data II

Red triangles: $\delta$ from 2 to 8; blue circles: $\delta$ from 20 to 80.

In this study, the SOHO EIT data set is shown to include two multifractal rank-ordered regimes, one for $\delta$ from 2 to 8 and the other for $\delta$ from 20 to 80.
APPLICATIONS TO SPACE PLASMAS

• Auroral electric field fluctuations (Tam et al.)
• Cusp and magnetosheath magnetic field fluctuations (Echim and Lamy)
• AE index (Consolini and De Michelis)
• EUV solar emissions (Wu and Uritsky)
• Solar wind (Podesta)
• Isotropic hydrodynamic turbulence (Wu)
• Cosmic Web (Chang et al.)
CUSP-Cluster data (Echim et al., 2007 and Lamy et al., 2008)
MODIFIED FOREST FIRE MODEL

Drossel and Schwabl, 1994
Loreto et al., 1995
Tam, Chang, Consolini, De Michelis, 2000
Rules of RG Transformation

At least one of the 4 sites with
Phase Space Global RG Trajectories