Introduction to

Space Plasma Complexity

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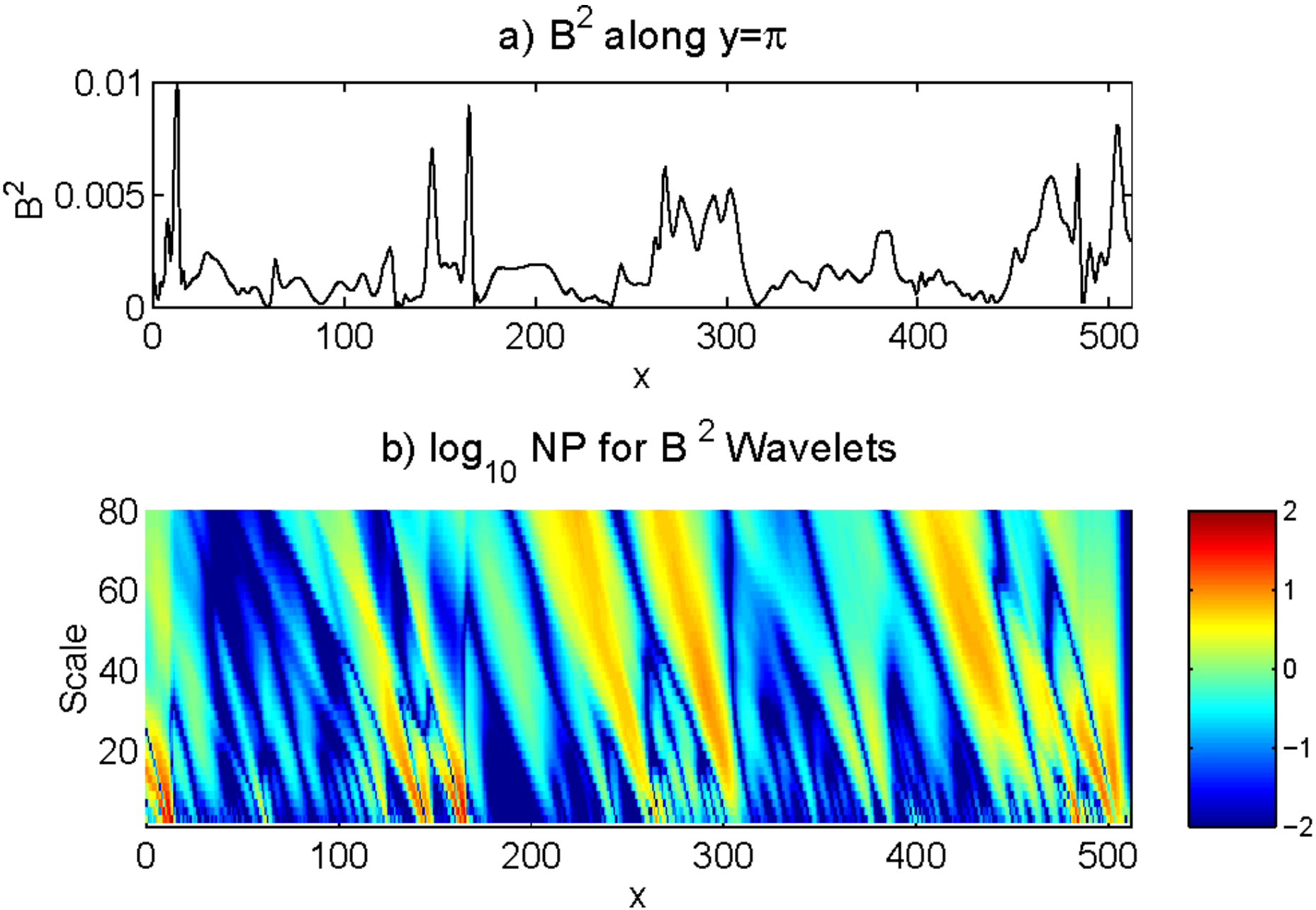

a) B^2 along y=π
0.01
0.005
0
B^2
0
100
200
300
400
500
x
b) log10 NP for B^2 Wavelets
80
60
40
20
Scale
0
100
200
300
400
500
x
2
1
0
−1
−2

ALFVÉN WAVES

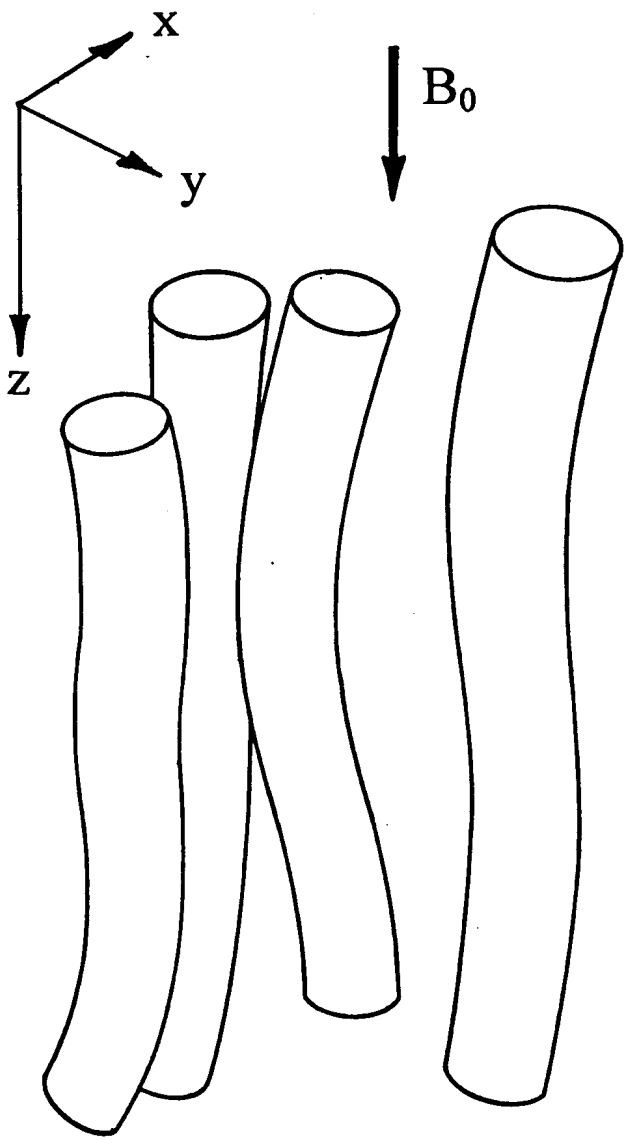
$$\frac{d\mathbf{B}}{dt} = (\mathbf{B} \bullet \nabla)\mathbf{V}$$

$$\rho \frac{d\mathbf{V}}{dt} = (\mathbf{B} \bullet \nabla)\mathbf{B} + \cdots$$

ALFVÉN RESONANCES

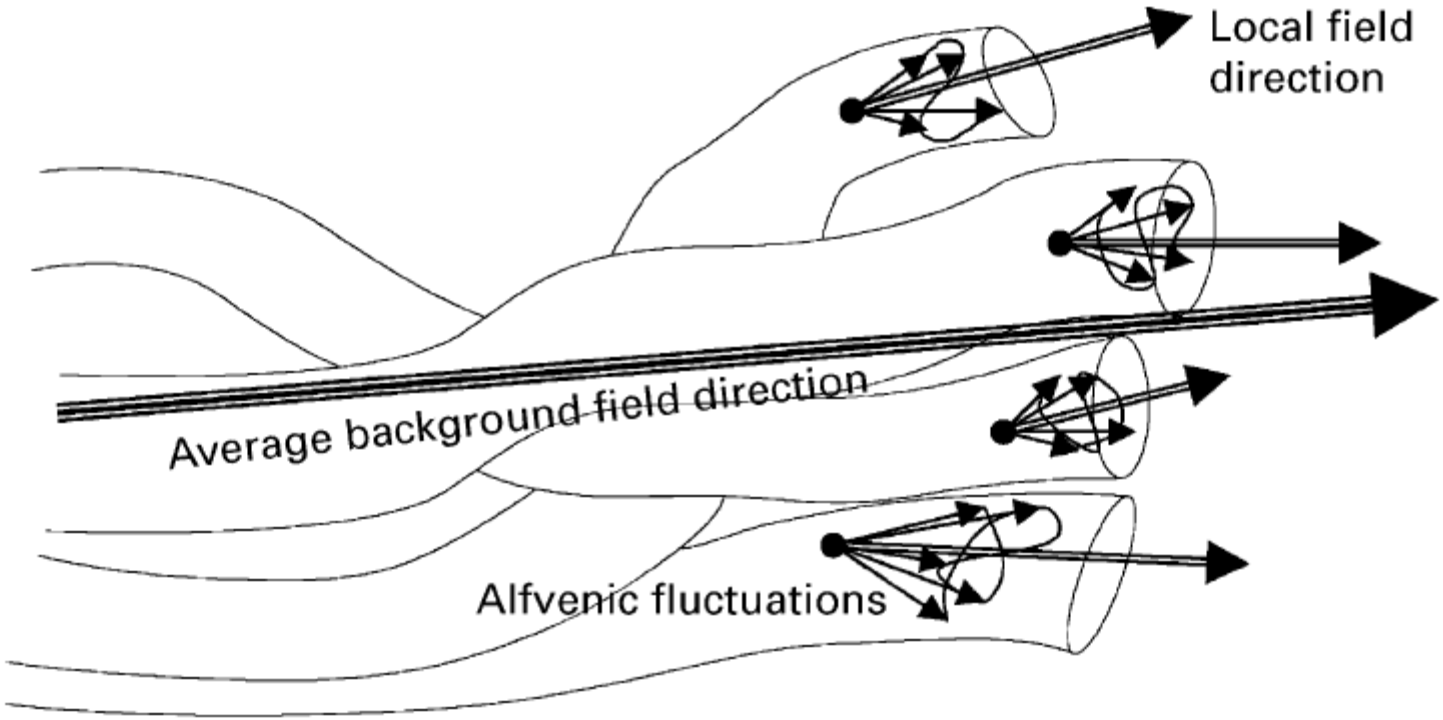
$$\mathbf{B} \bullet \nabla \rightarrow i\mathbf{k} \bullet \mathbf{B} = 0$$

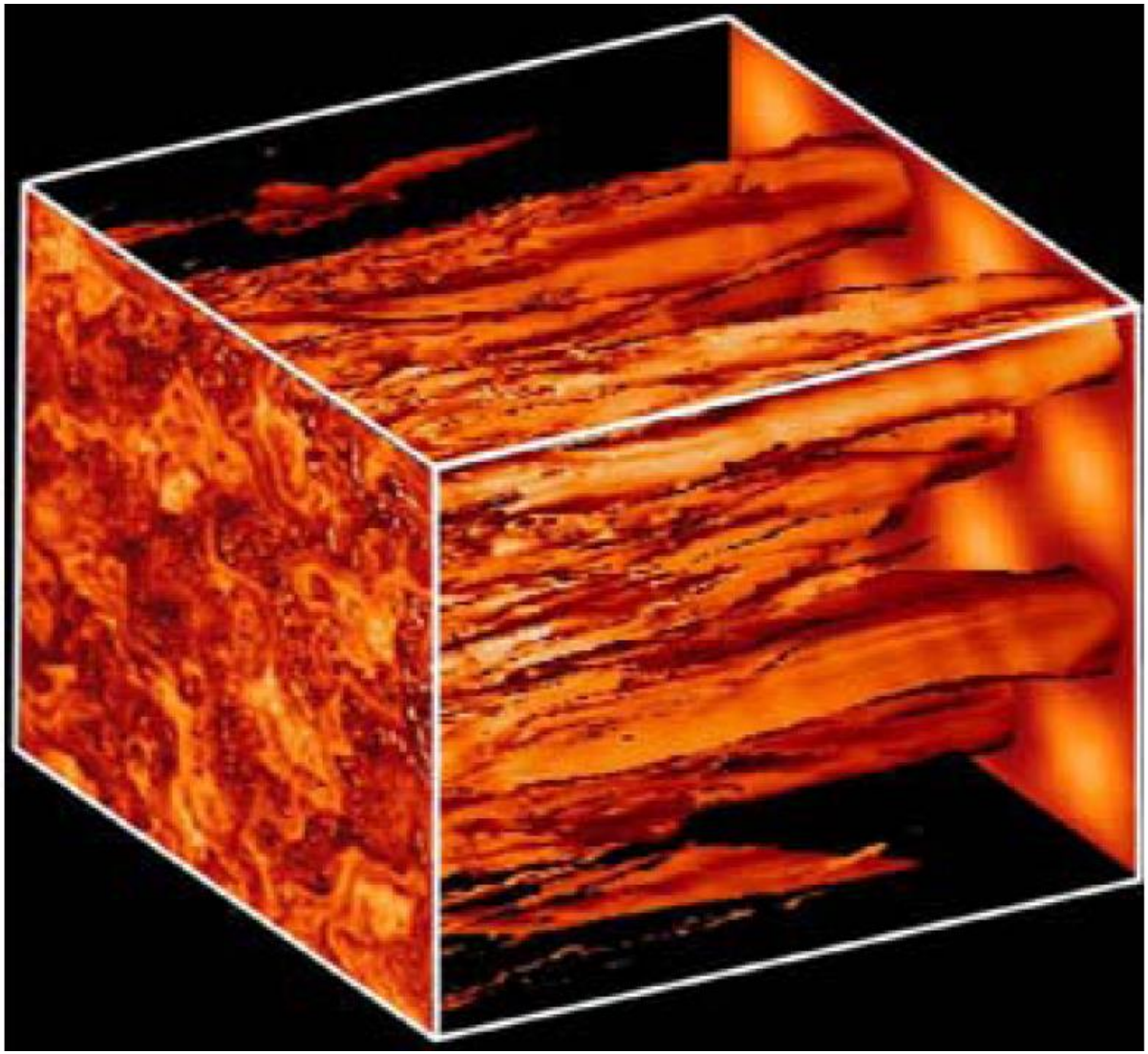
SOLAR WIND



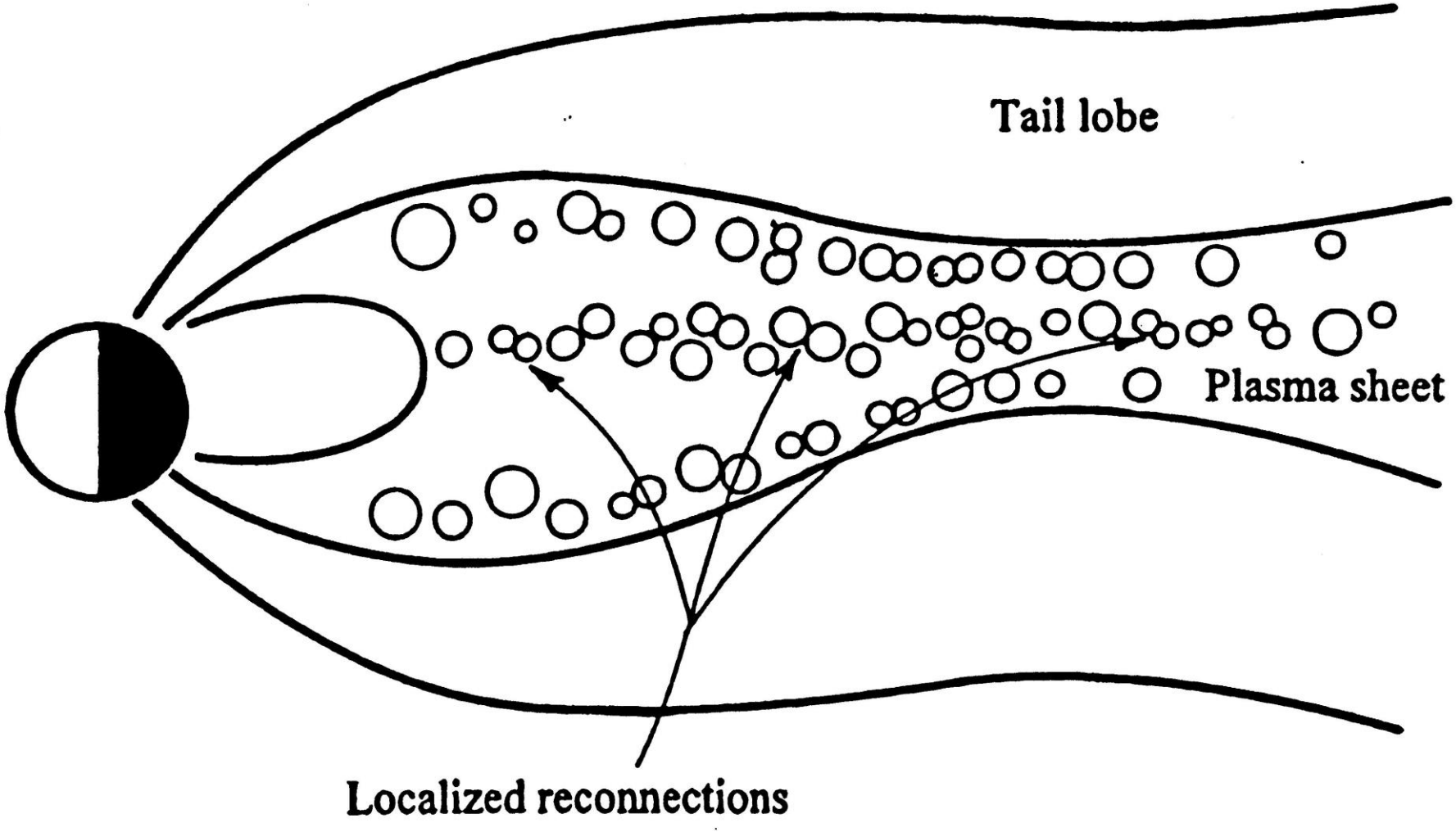
CHANG (1998)

BRUNO ET AL. (2001)


Local field
direction
Average background field direction
Alfvenic fluctuations

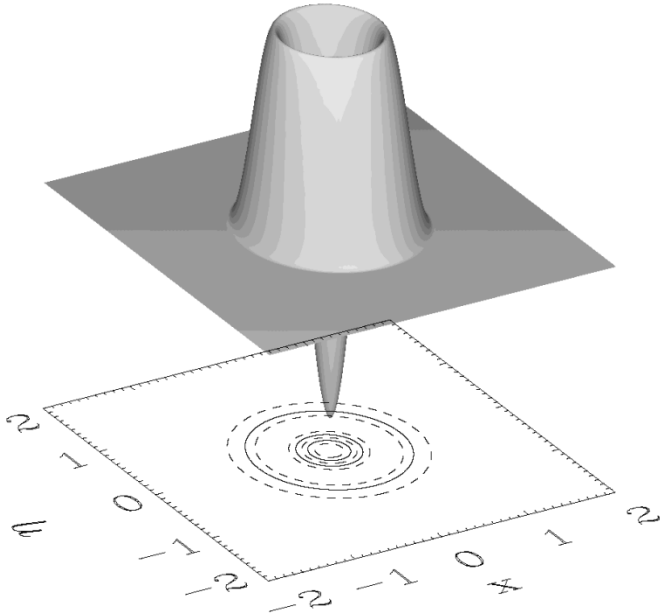


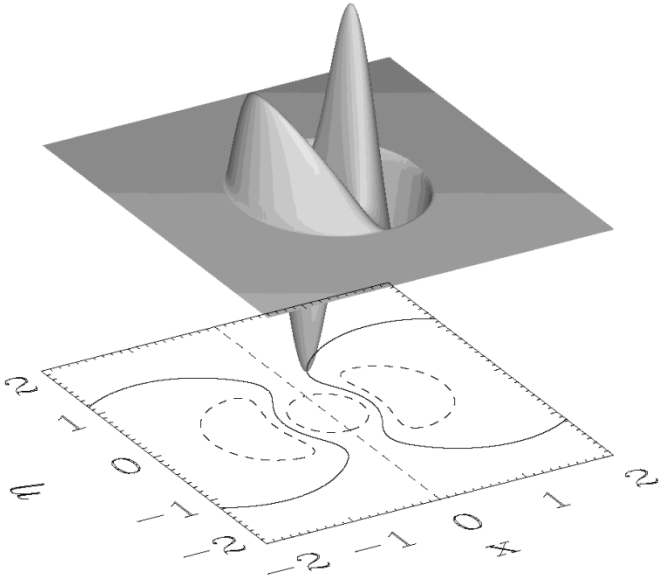
Matthaeus, Qin, Bieber, and Zank, *Astrophys. J. Lett.* 2003

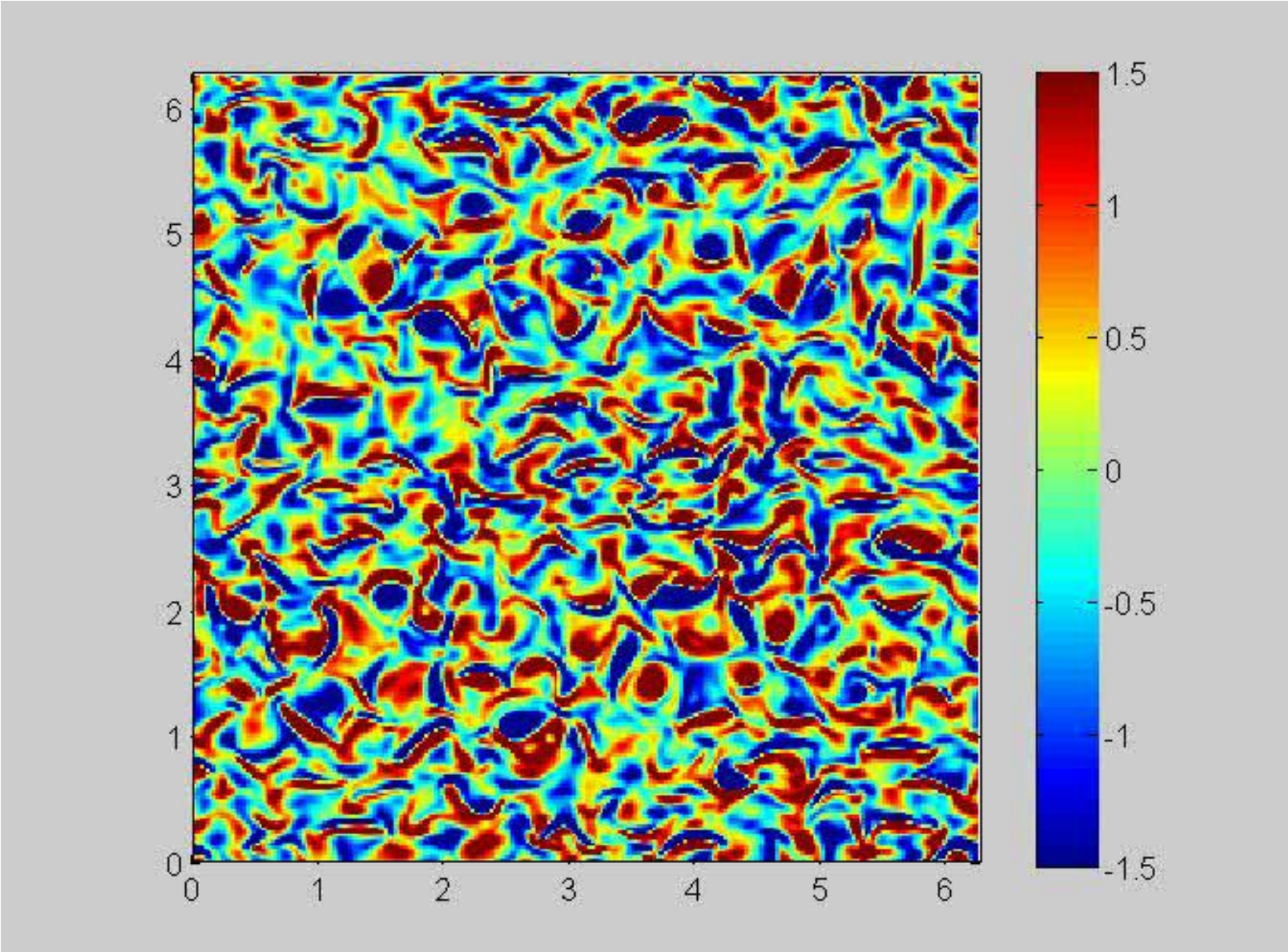


CHANG, 1998

ALEXANDROVA 2008

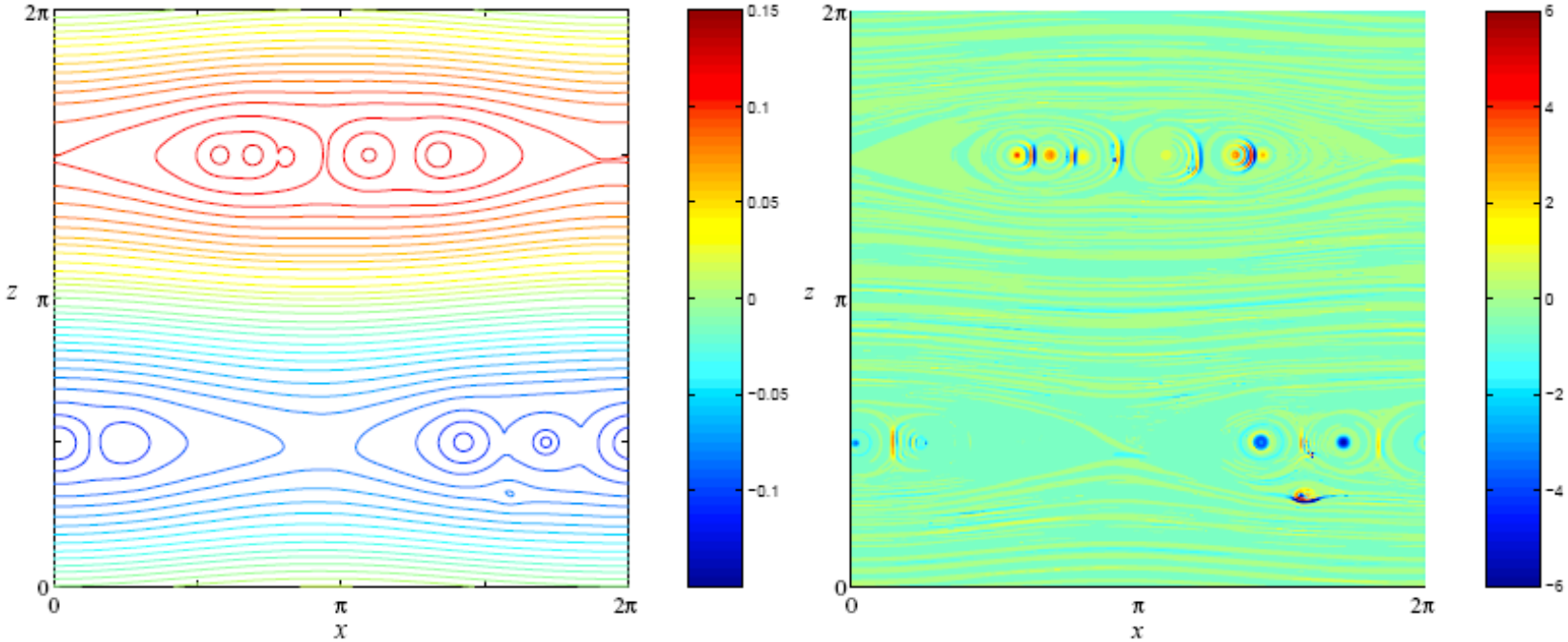






WU & CHANG, 2006

(c) $t = 600$

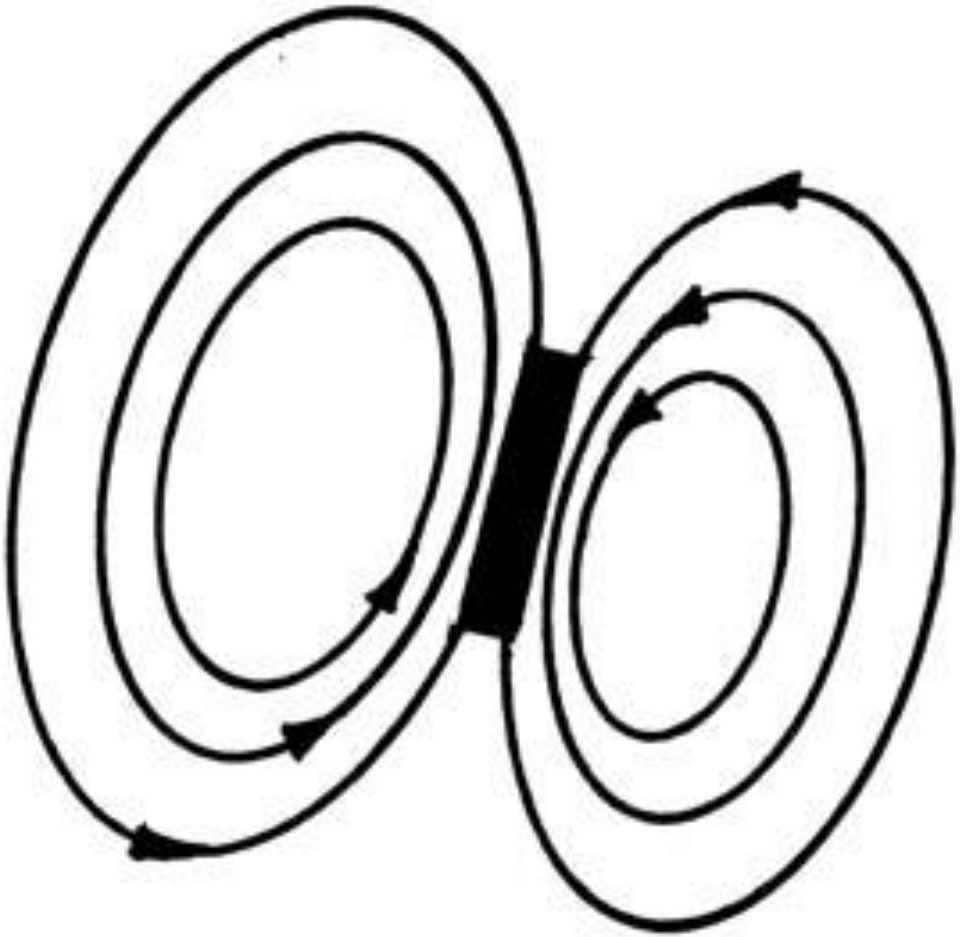


Coarse-Grained Dissipation

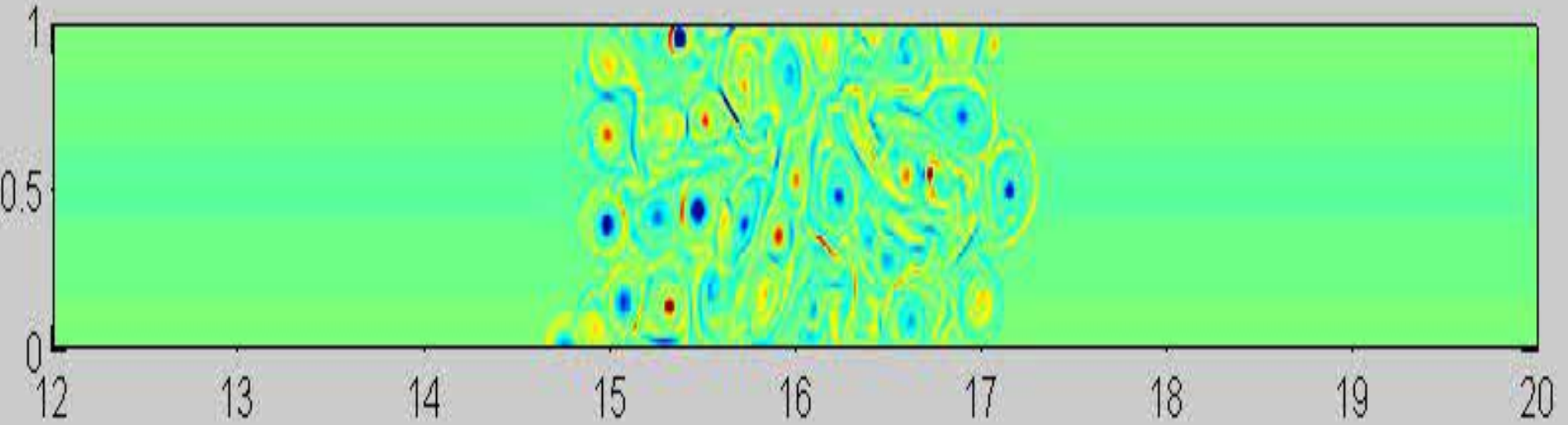
And

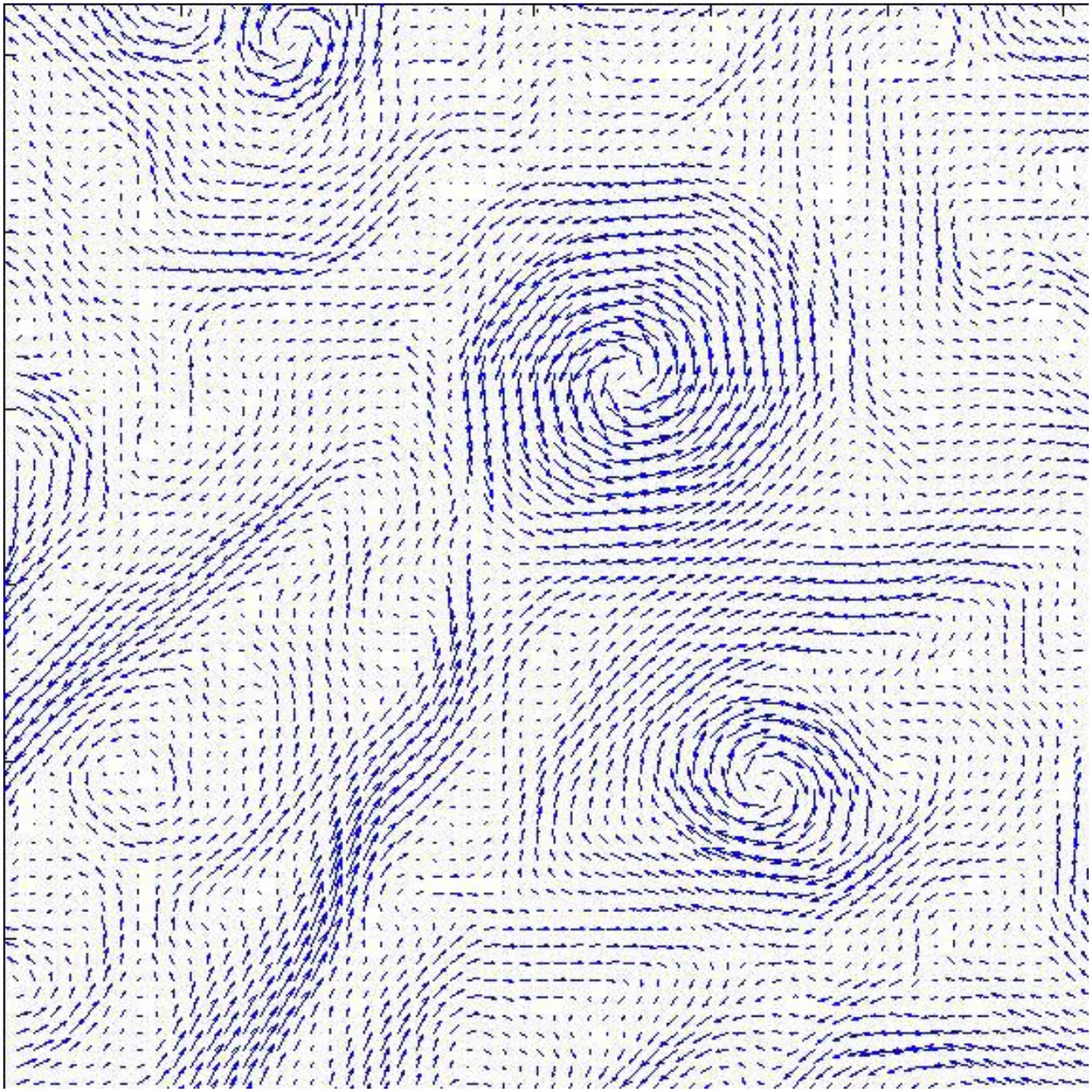
Magnetic Reconfiguration

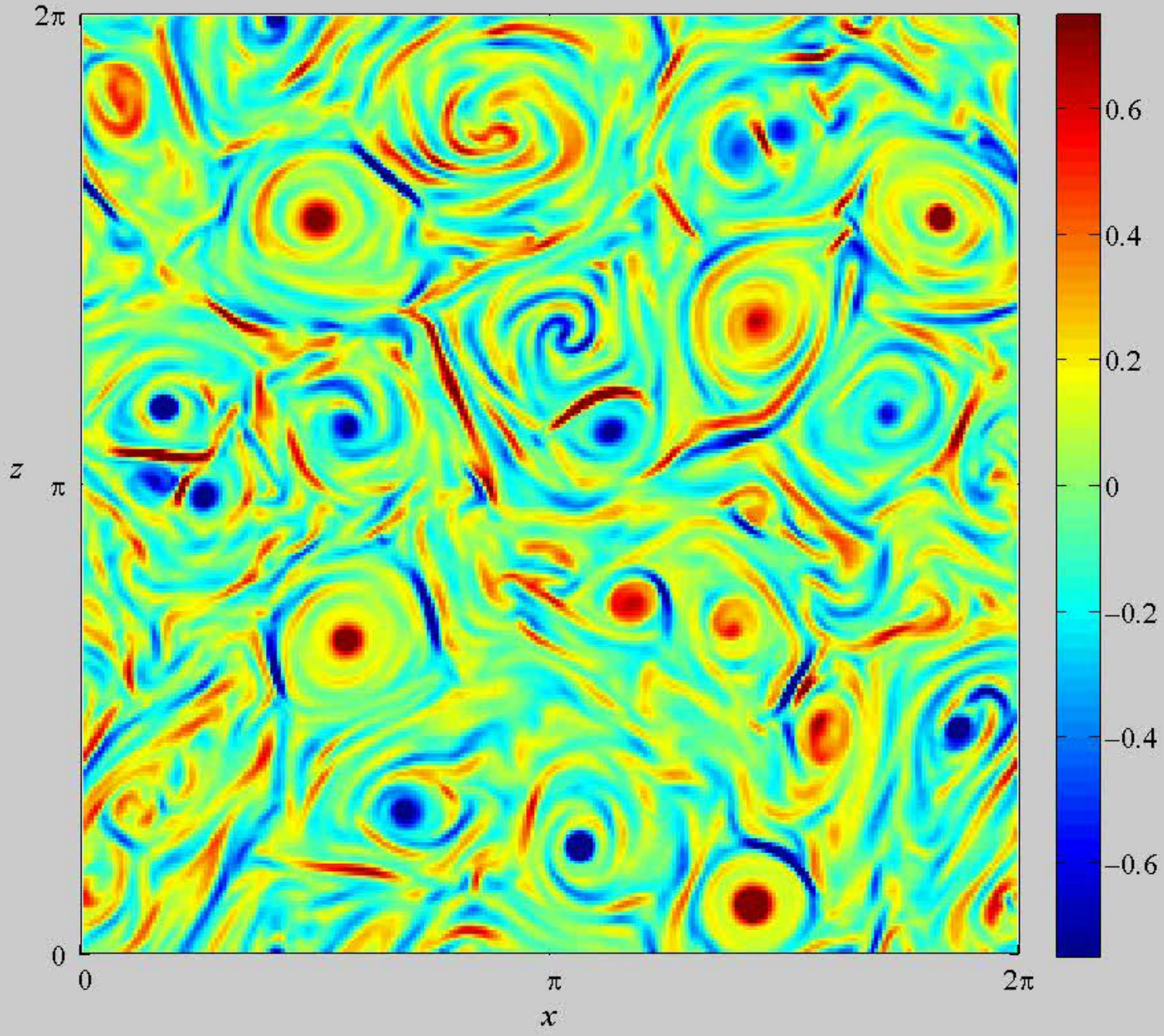
WU & CHANG, 2001

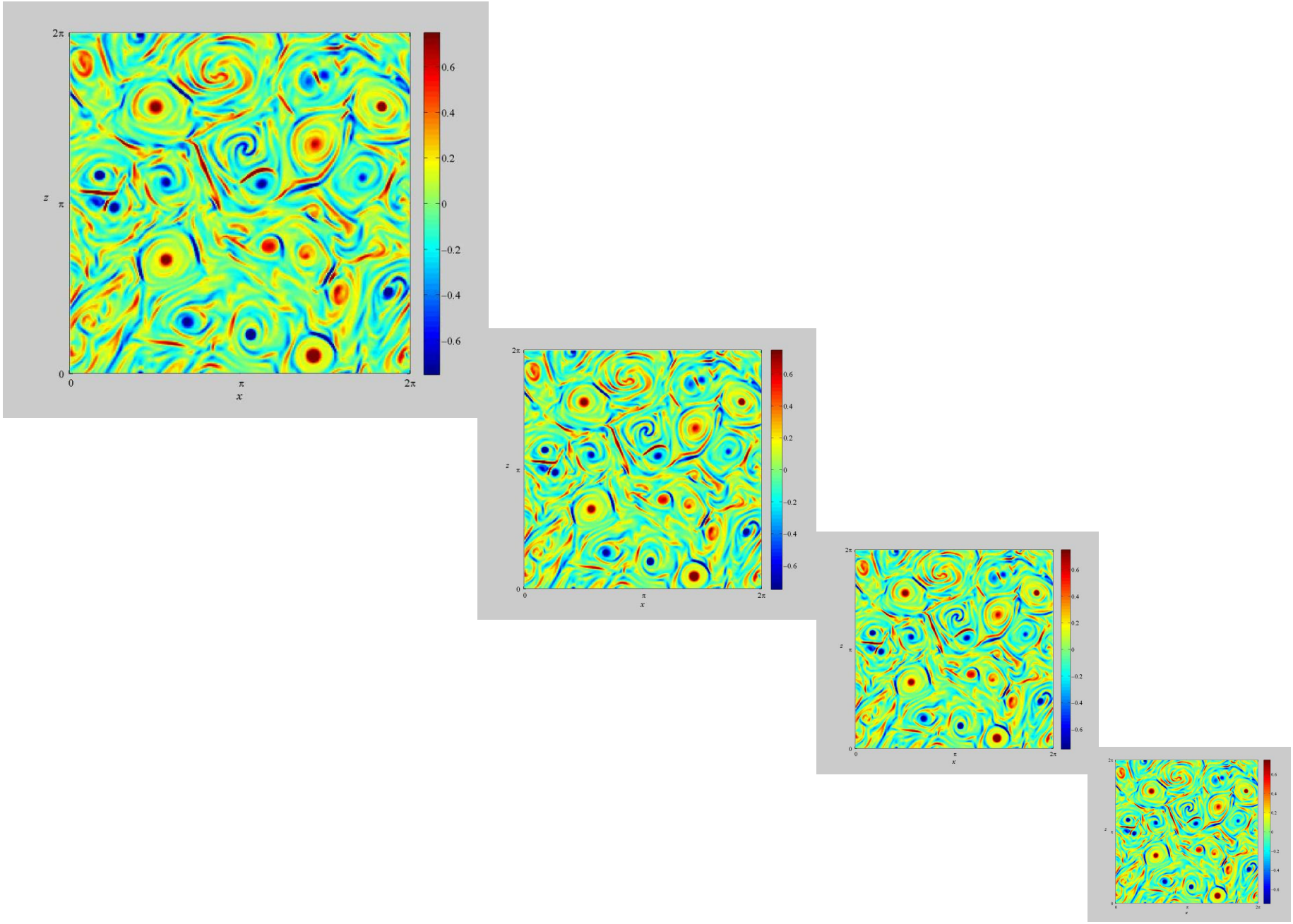


COARSE-GRAINED DISSIPATION






2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6


2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6

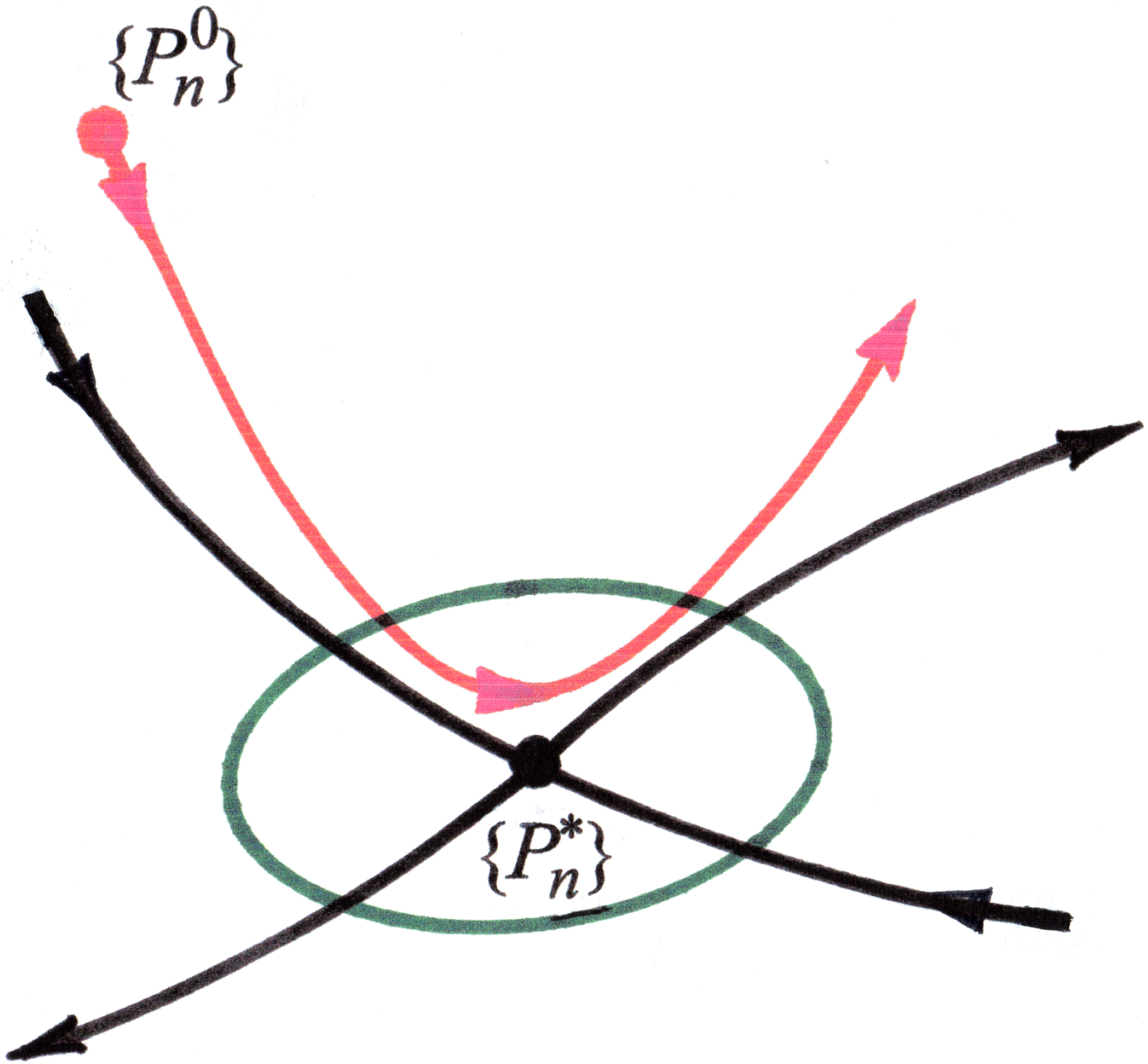
PARAMETER SET: $\{P\}$

DRG TRANSFORMATION:

$$d\{P\}/dl = \mathcal{R}\{P\}$$

Procedure:

Search for Invariants


$\{P_n^0\}$
$\{P_n^*\}$

LINEAR TRANSFORMATIONS

$\rightarrow$ POWER LAW INVARIANTS.

****

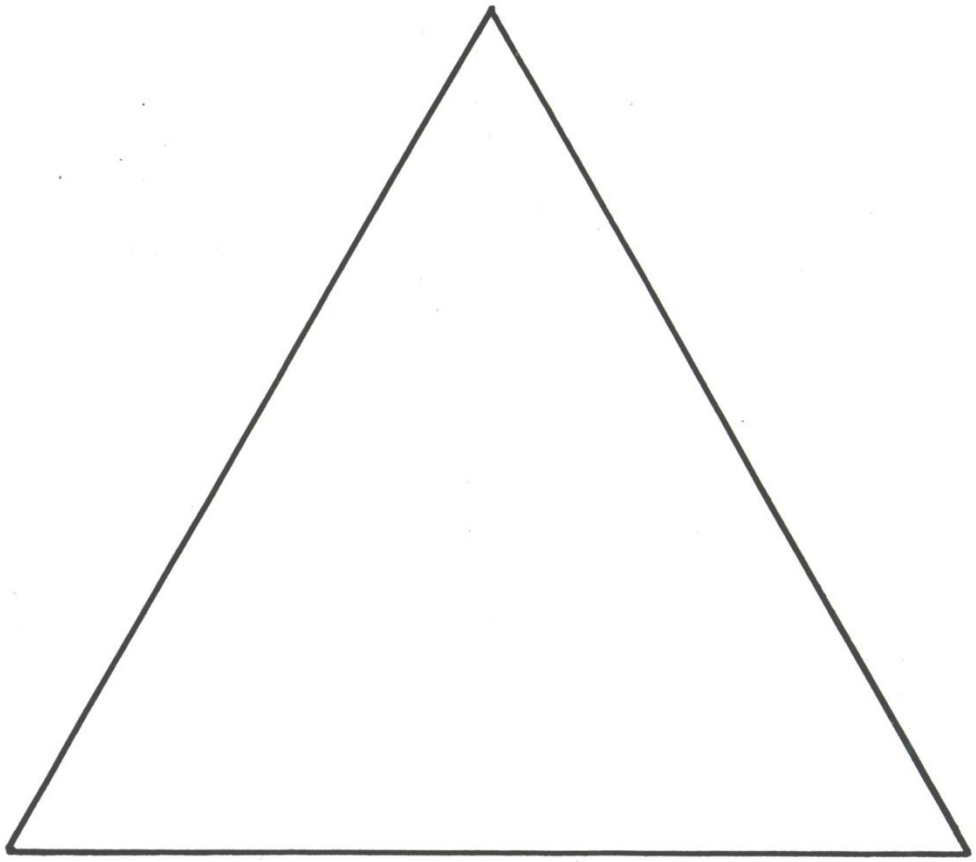
<u>*EXAMPLE:*</u>

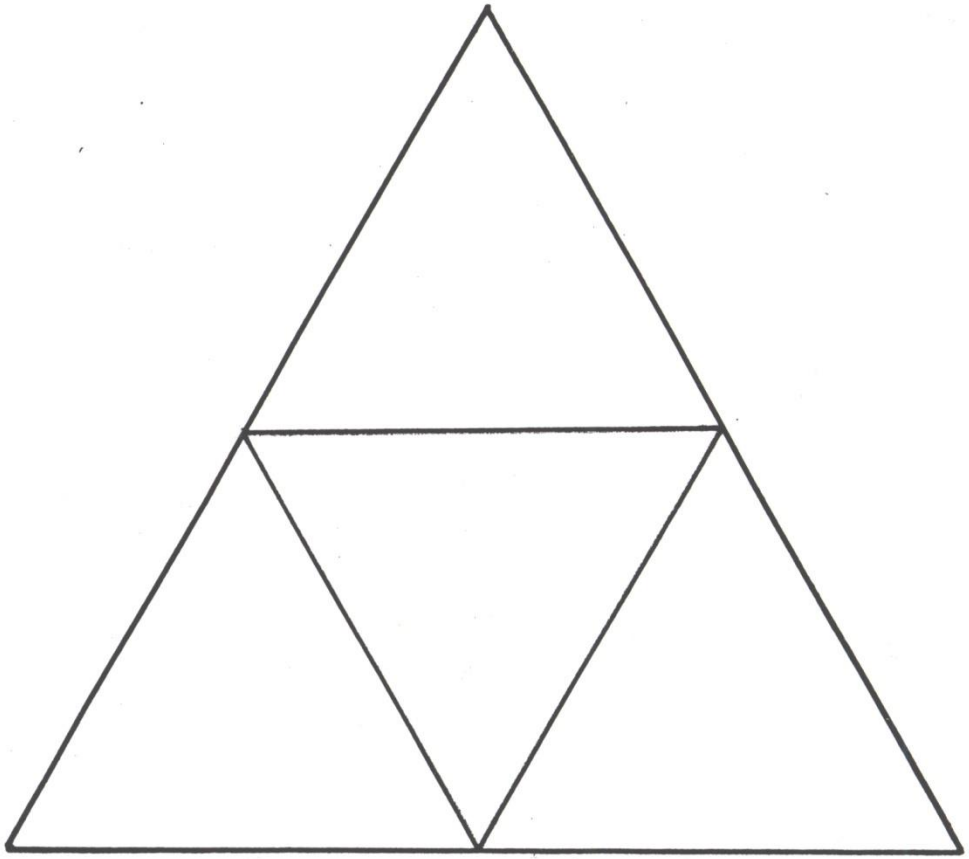
$P \rightarrow e^{a_P} P, \quad E \rightarrow e^{a_E} E$ *LEADS TO*

$P / E^{a_P / a_E}$ AS AN INVARIANT.

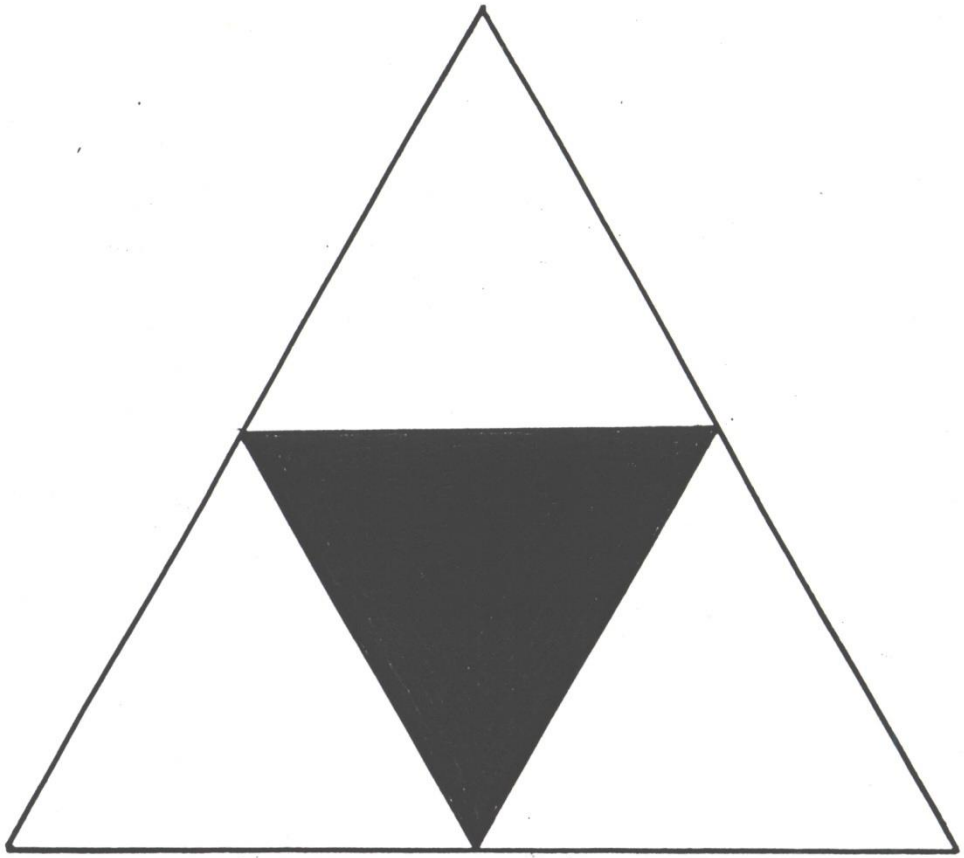
HOW DO WE DEFINE

DIMENSIONS?

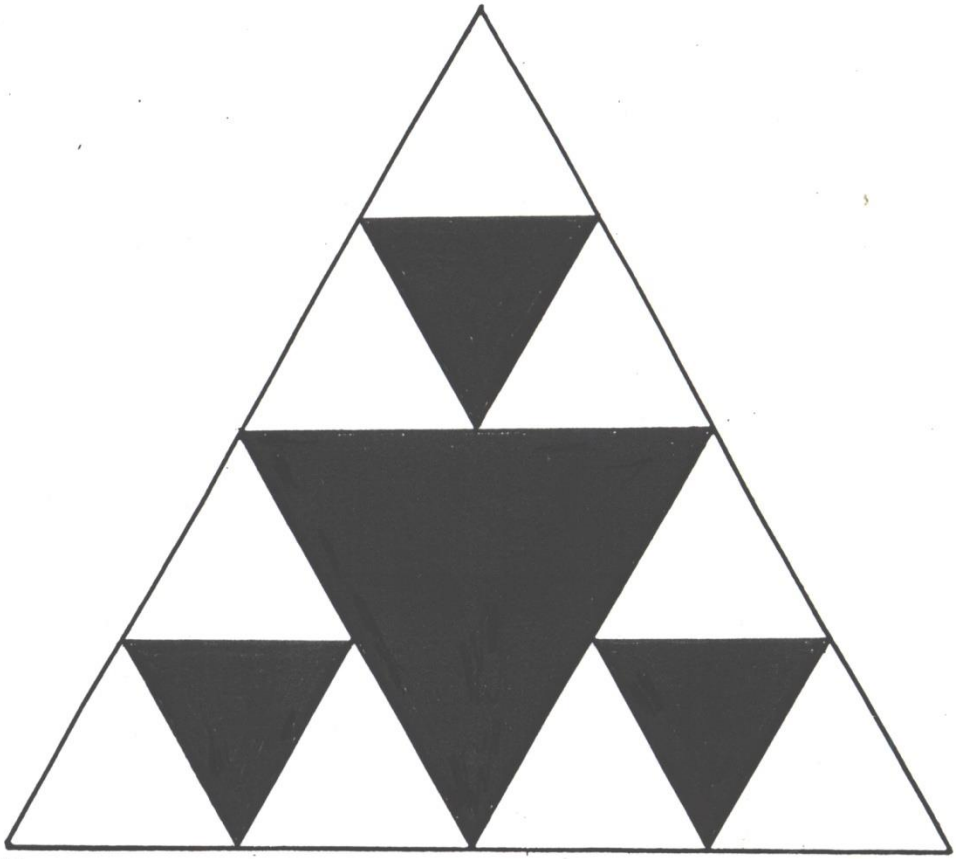




$$d = \log 4 / \log 2 = 2$$

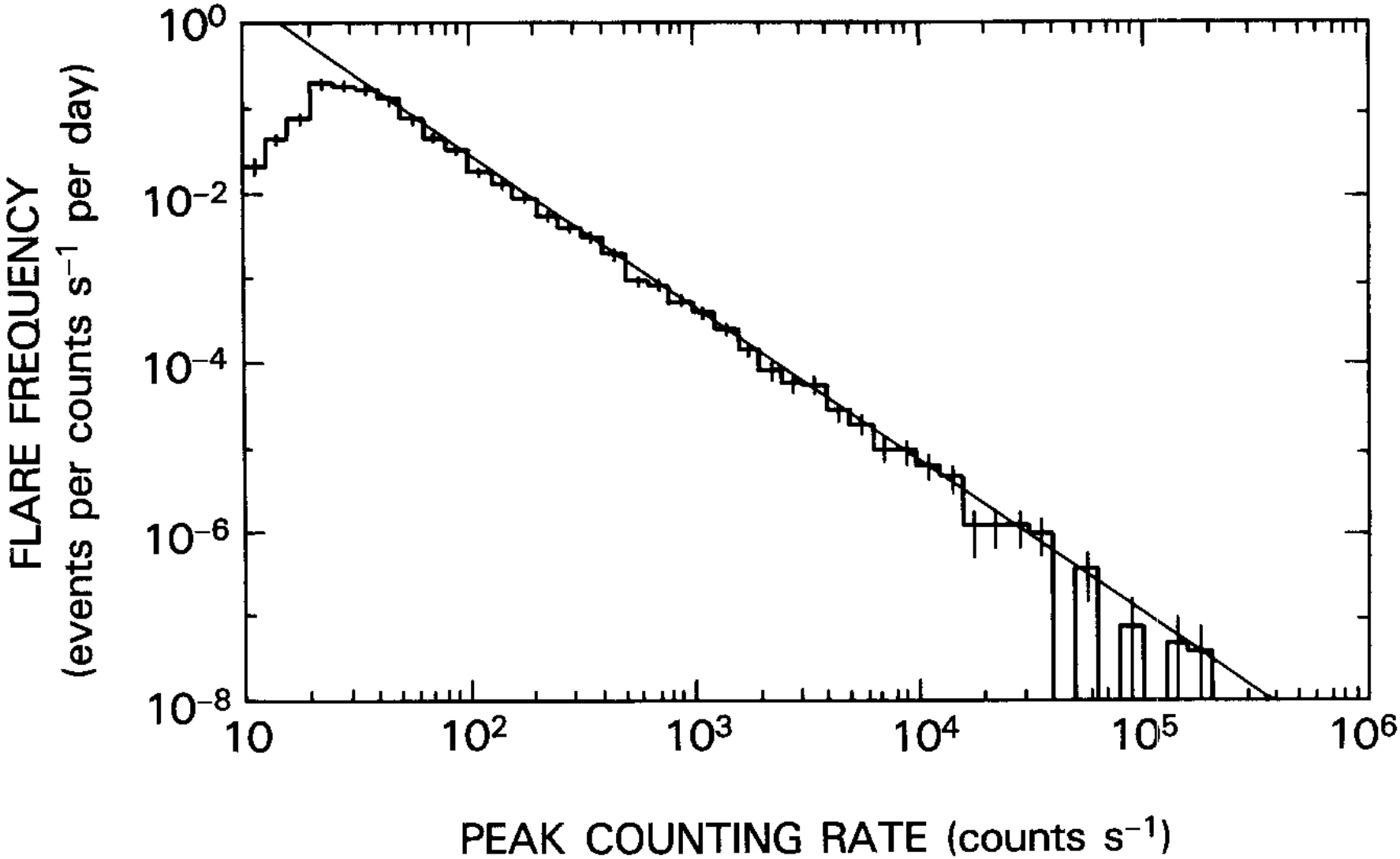


SIERPINSKI TRIANGLE (1916)



$$d = \log 3 / \log 2 \approx 1.58$$

Dennis, 1985


FLARE FREQUENCY
(events per counts s^{-1} per day)
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10
10^2
10^3
10^4
10^5
10^6
PEAK COUNTING RATE (counts s^{-1})

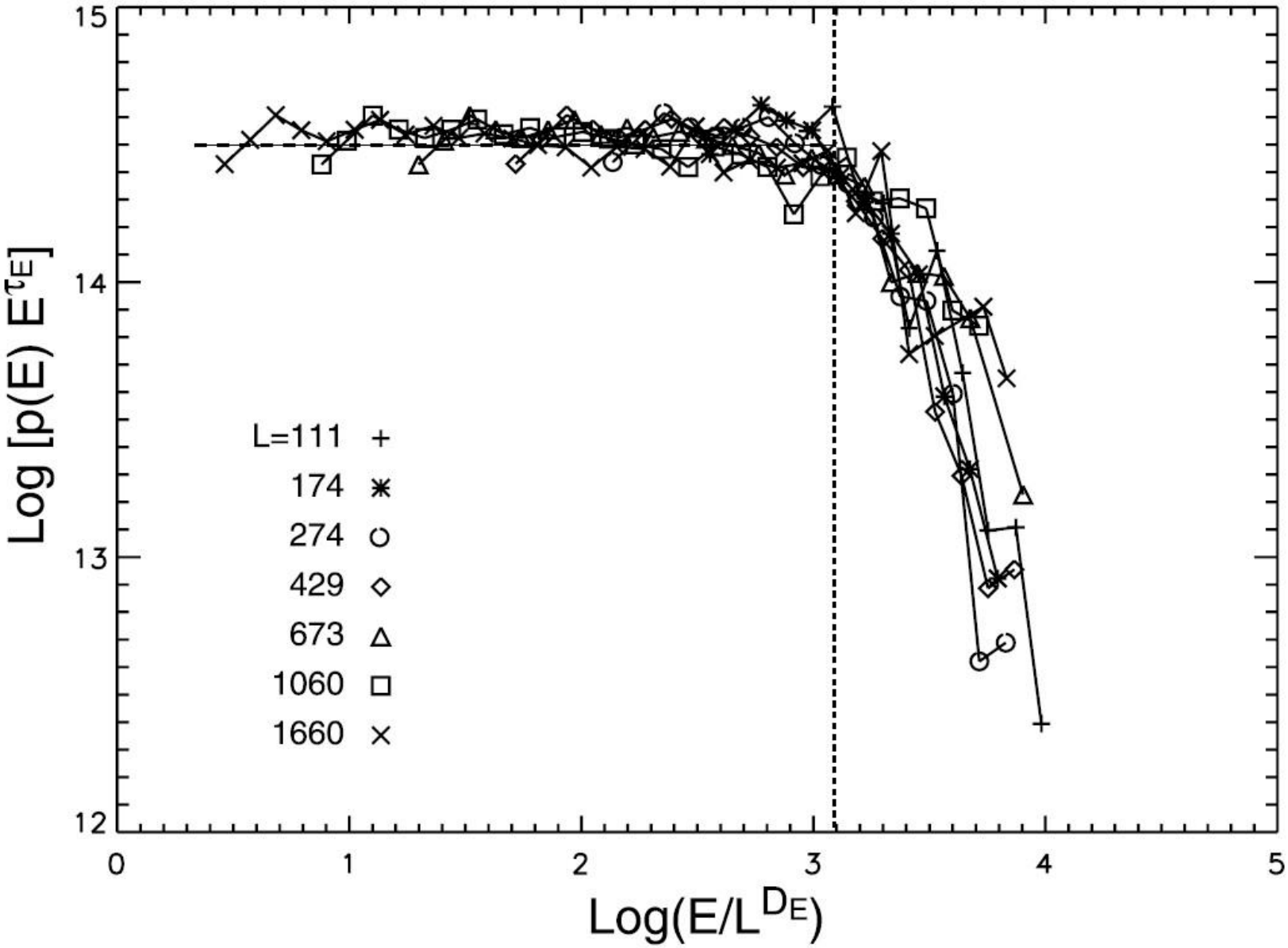
$$P(E,L)$$

$$P/E^a = I$$

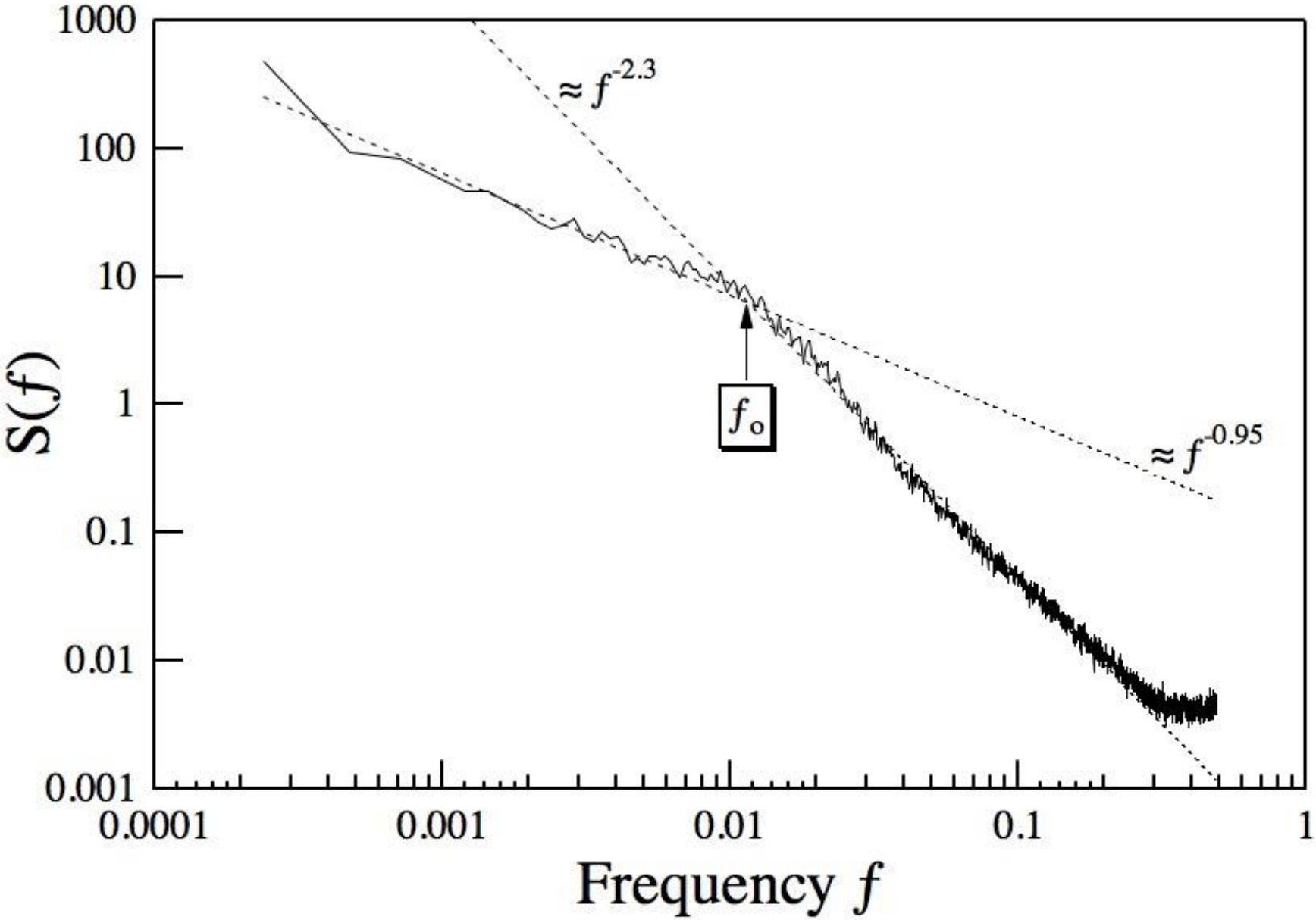
$$E/L^b = J$$

$$\rightarrow I = F(J)$$

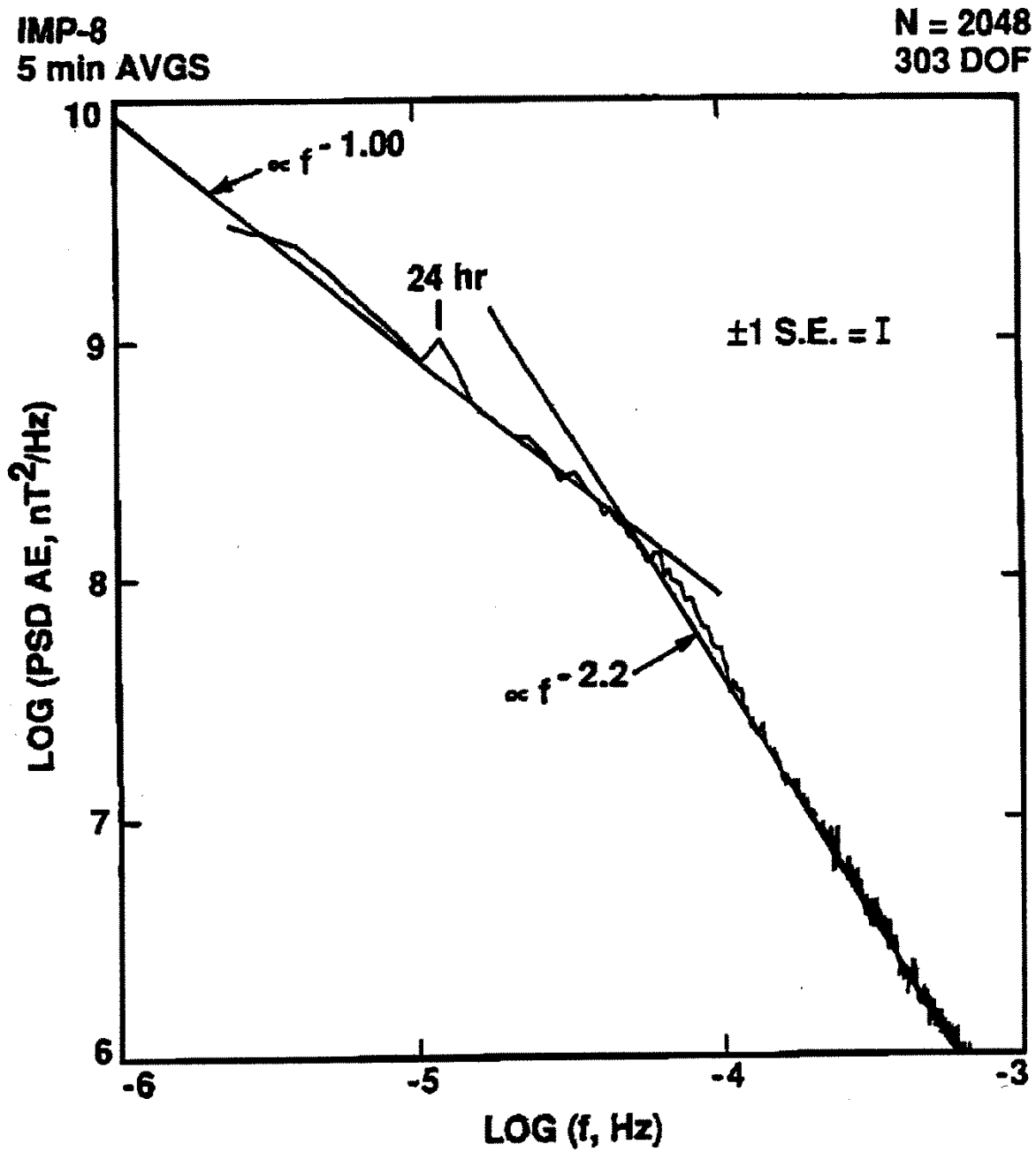
Finite Size Scaling (FSS)


15
14
13
12
0
1
2
3
4
5
Log [p(E) E^τE]
Log(E/L^DE)
L=111
174
274
429
673
1060
1660

a.


1000
100
10
1
0.1
0.01
0.001
S(f)
0.0001
0.001
0.01
0.1
1
Frequency f
≈ f^-2.3
≈ f^-0.95
f_o

1978 - 1980 AE

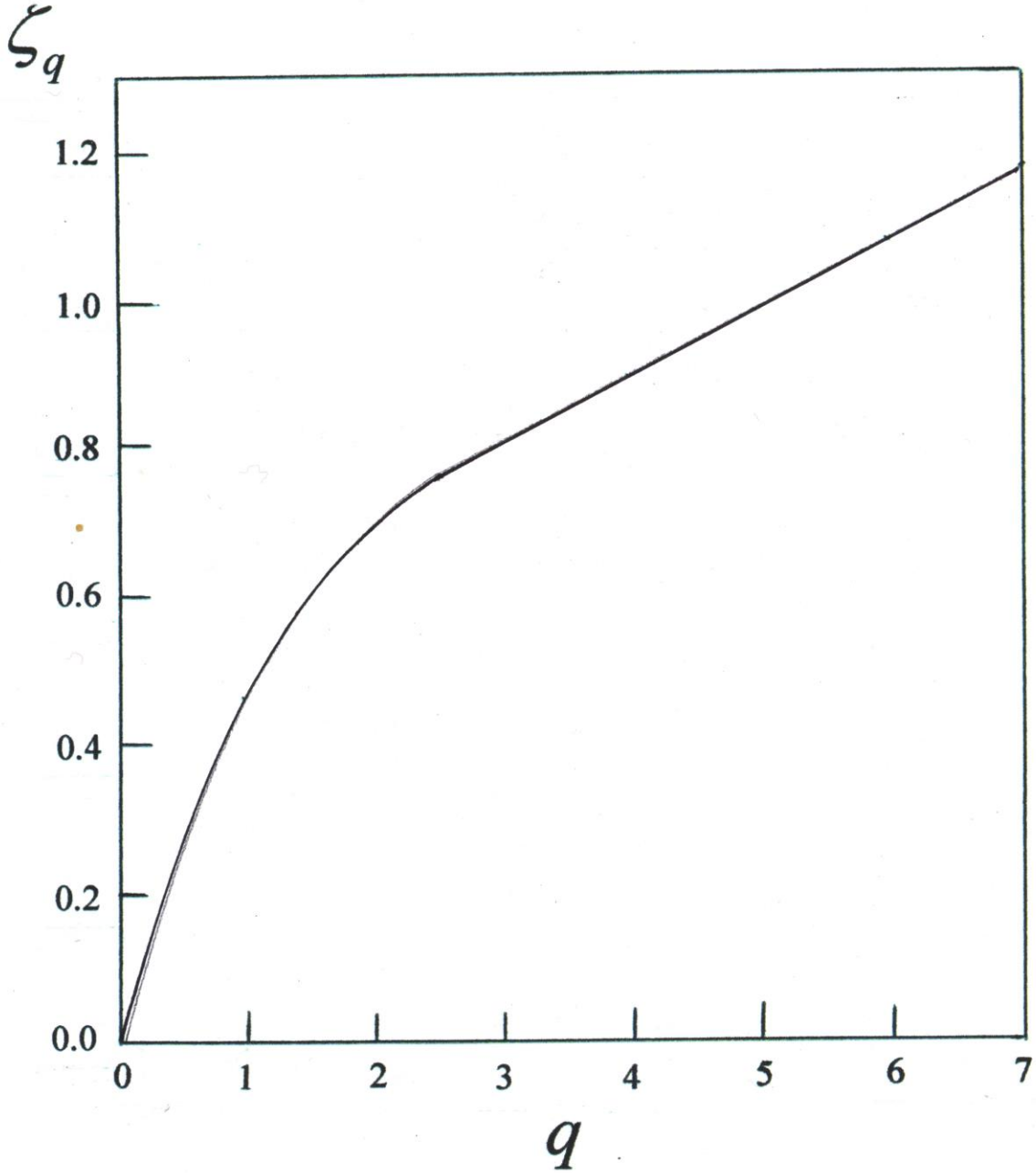

IMP-8
5 min AVGS
N = 2048
303 DOF
∝ f^-1.00
24 hr
±1 S.E. = I
∝ f^-2.2
LOG (PSD AE, nT²/Hz)
LOG (f, Hz)
10
9
8
7
6
-6
-5
-4
-3

COARSE-GRAINED MEASURE:

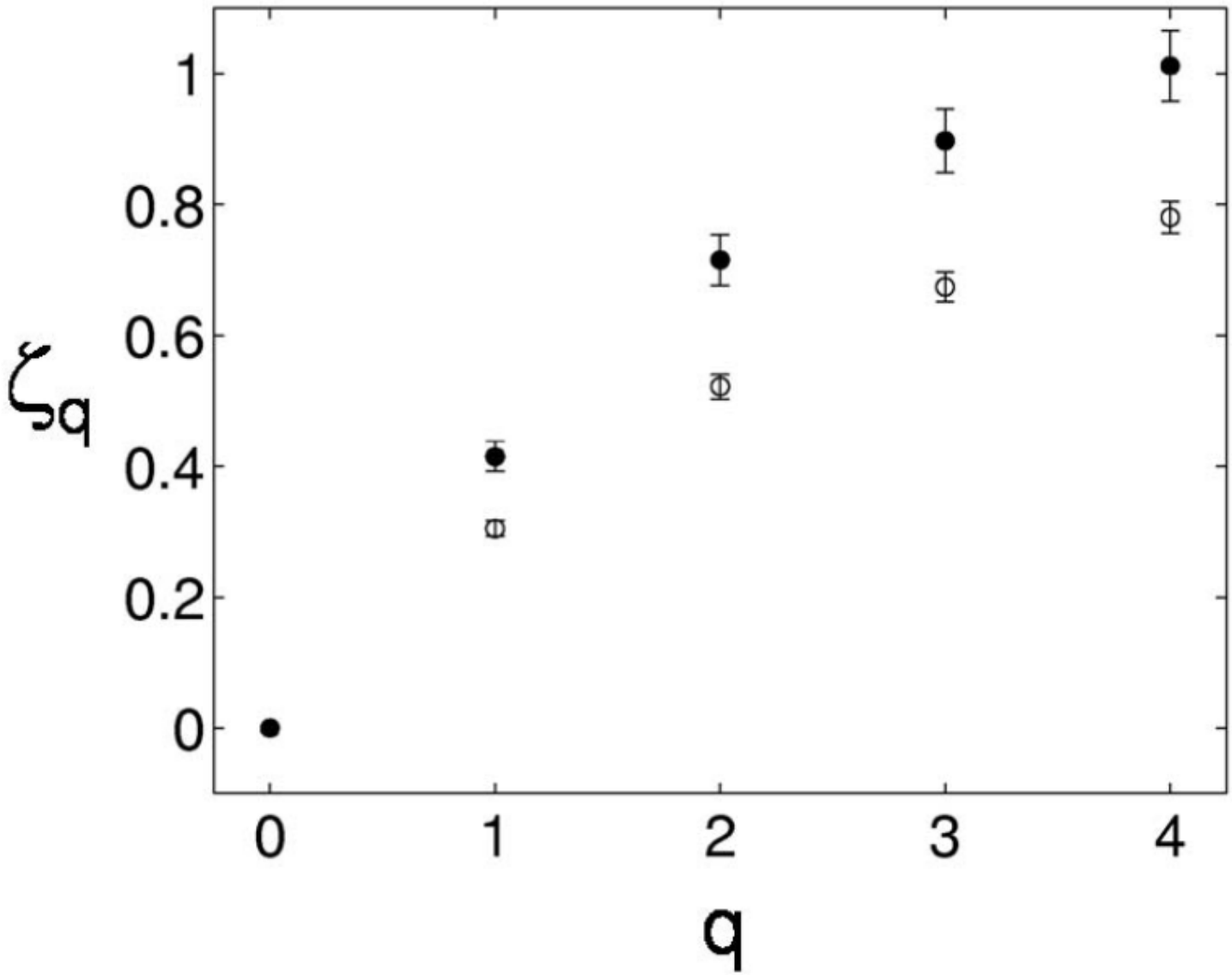
STRUCTURE FUNCTIONS

$$S_q \equiv \left\langle \left| B(x_i + \Delta) - B(x_i) \right|^q \right\rangle \sim \Delta^{\zeta_q}, \qquad \Delta = k\delta$$

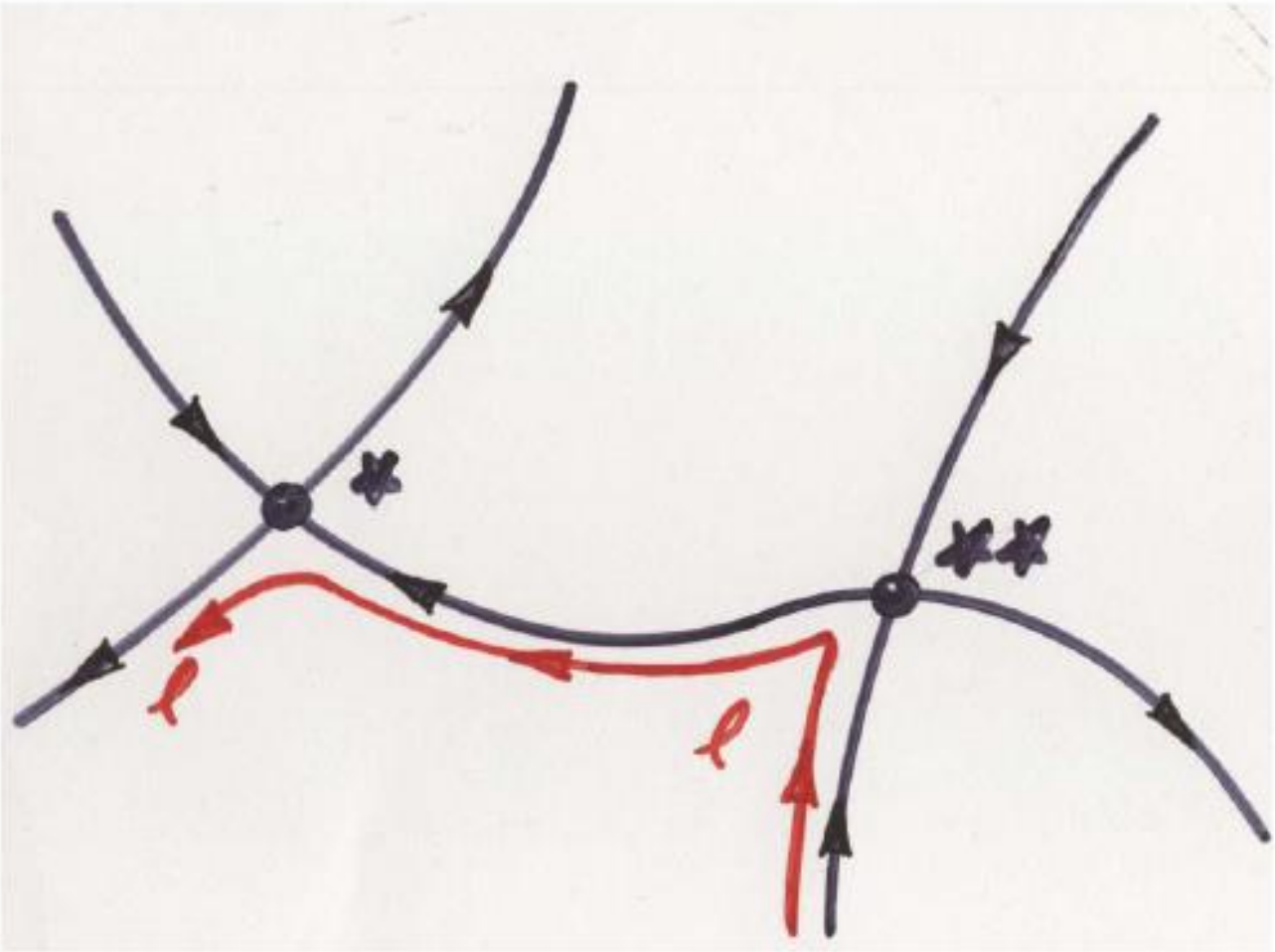
MULTI-FRACTALS



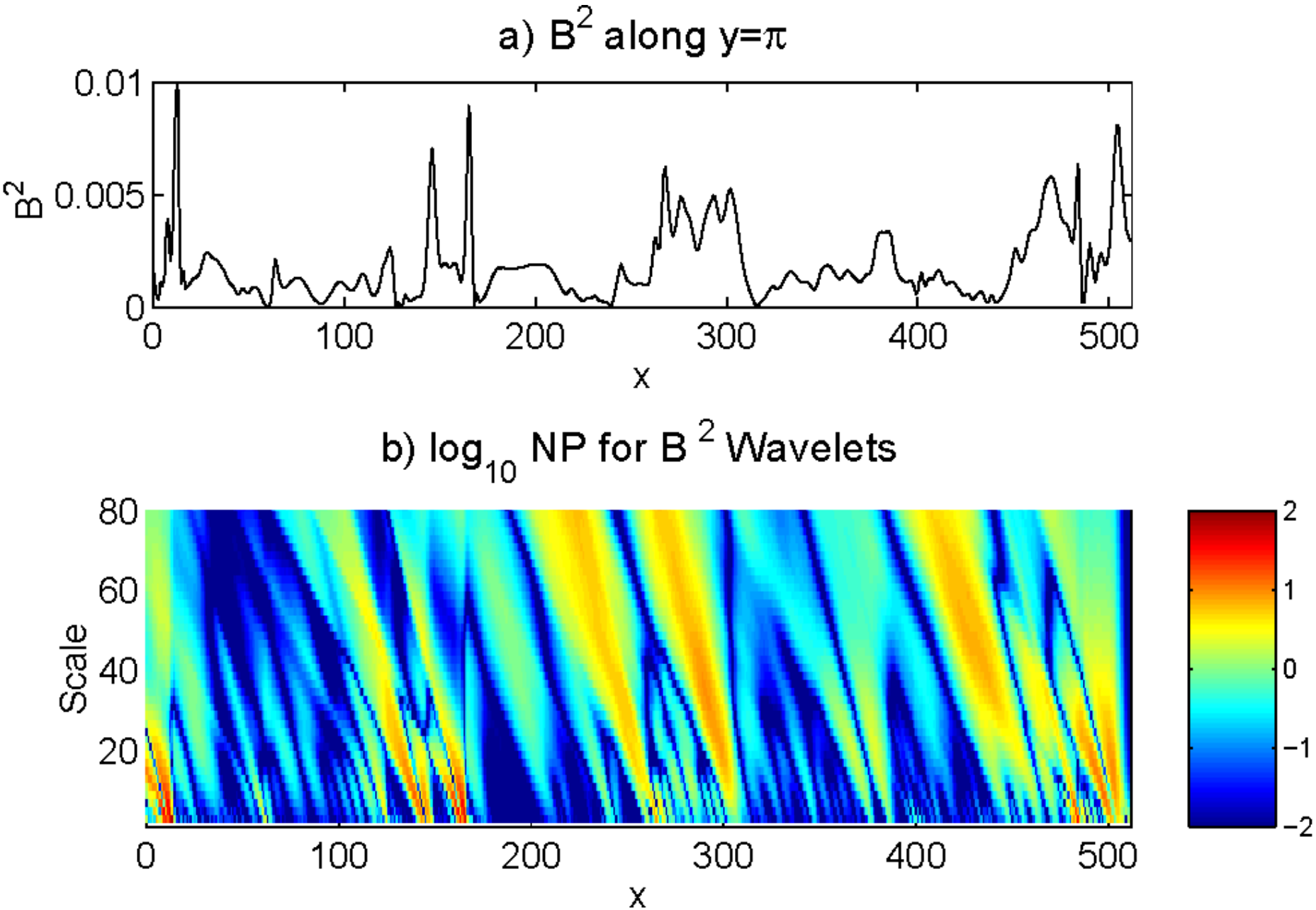
Bruno et al. 2007, Magnetic field strength fluctuations

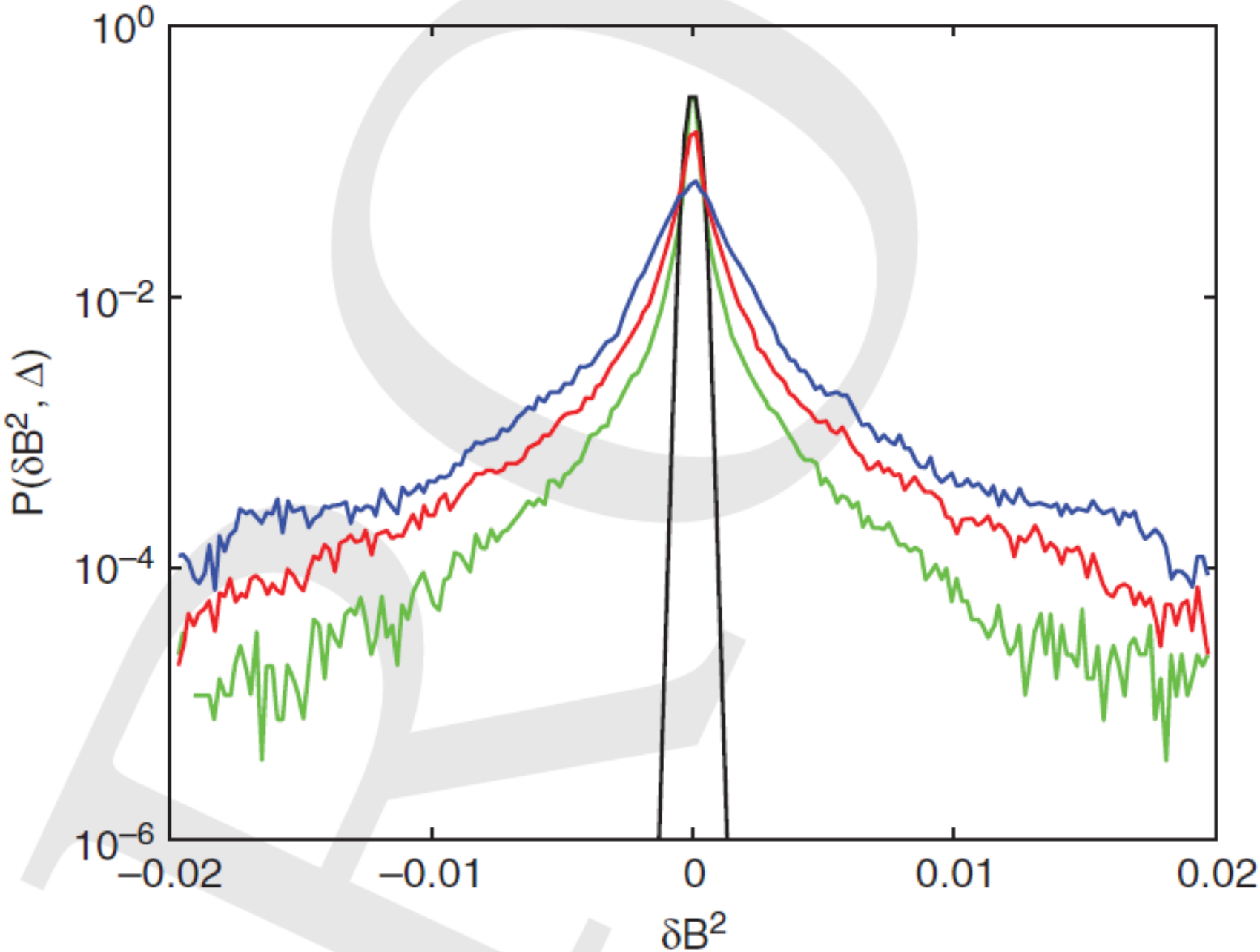


$\{P_N\}$



Fixed Points: $\dfrac{d\{P_N\}}{d\ell}=0$


a) B^2 along y=π
0.01
0.005
0
B^2
0
100
200
300
400
500
x
b) log10 NP for B^2 Wavelets
80
60
40
20
Scale
0
100
200
300
400
500
x
2
1
0
−1
−2


10^0
10^{-2}
10^{-4}
10^{-6}
$P(\delta B^2, \Delta)$
−0.02
−0.01
0
0.01
0.02
δB^2

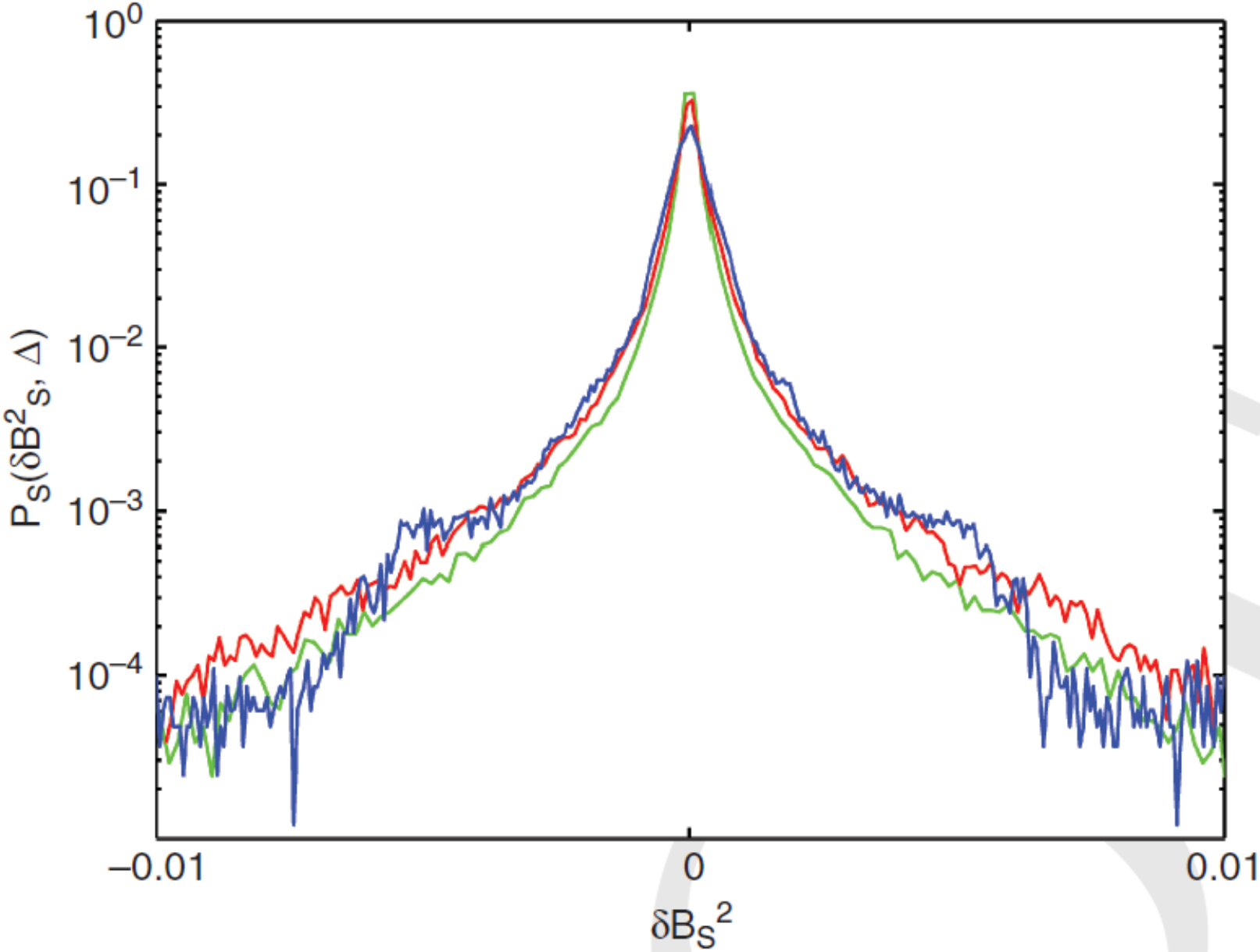
PROBABILITY DISTRIBUTION FUNCTION OF INCREMENTAL FLUCTUATIONS

$$P(\delta B^2, \Delta)$$

SCALING NEAR DYNAMIC CRITICALITY

$$P(\delta B^2, \Delta)\Delta^{-s} = P_s(\delta B^2{}_S)$$

$$\delta B^2{}_S = \delta B^2 / \Delta^s$$

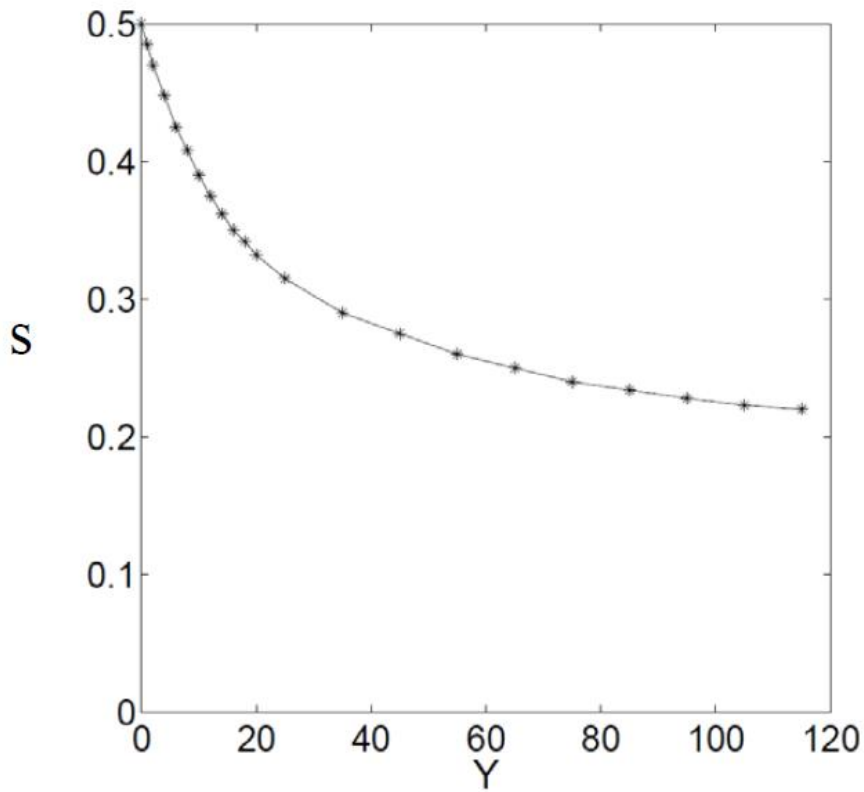


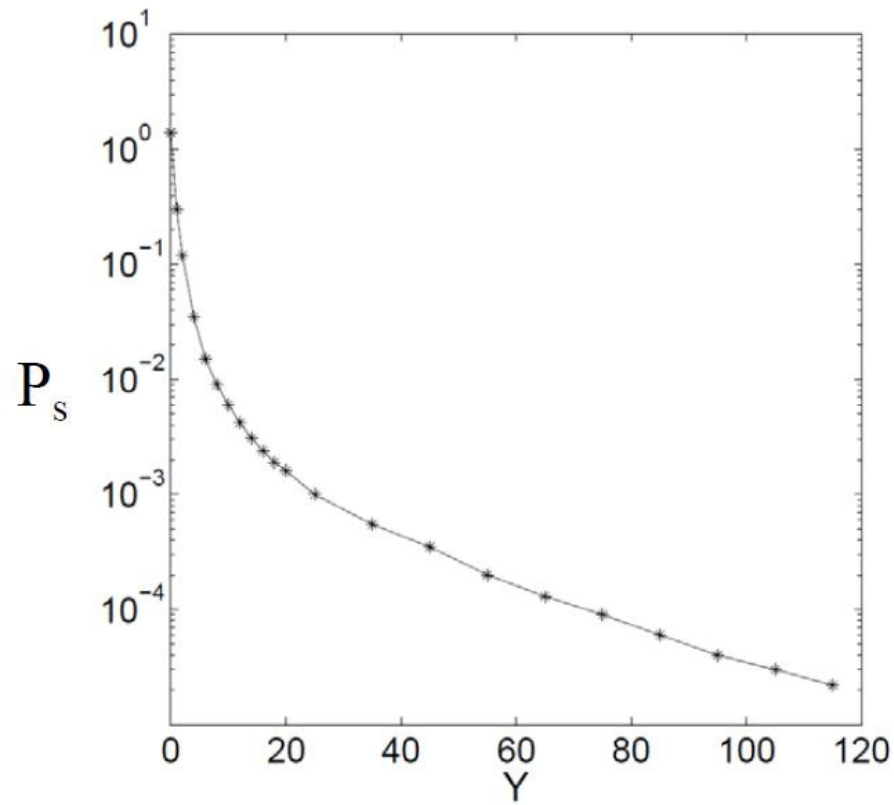
Hnat et al., 2002 (Solar Wind)

Chang et al., 2004

$$P(\delta B^2,\Delta)\Delta^{-s} = P_s(\delta B^2{}_S)$$

$$\mathrm{Y} = \delta B^2{}_S = \delta B^2 / \Delta^s \qquad \mathrm{S}=\mathrm{S}(\mathrm{Y})$$

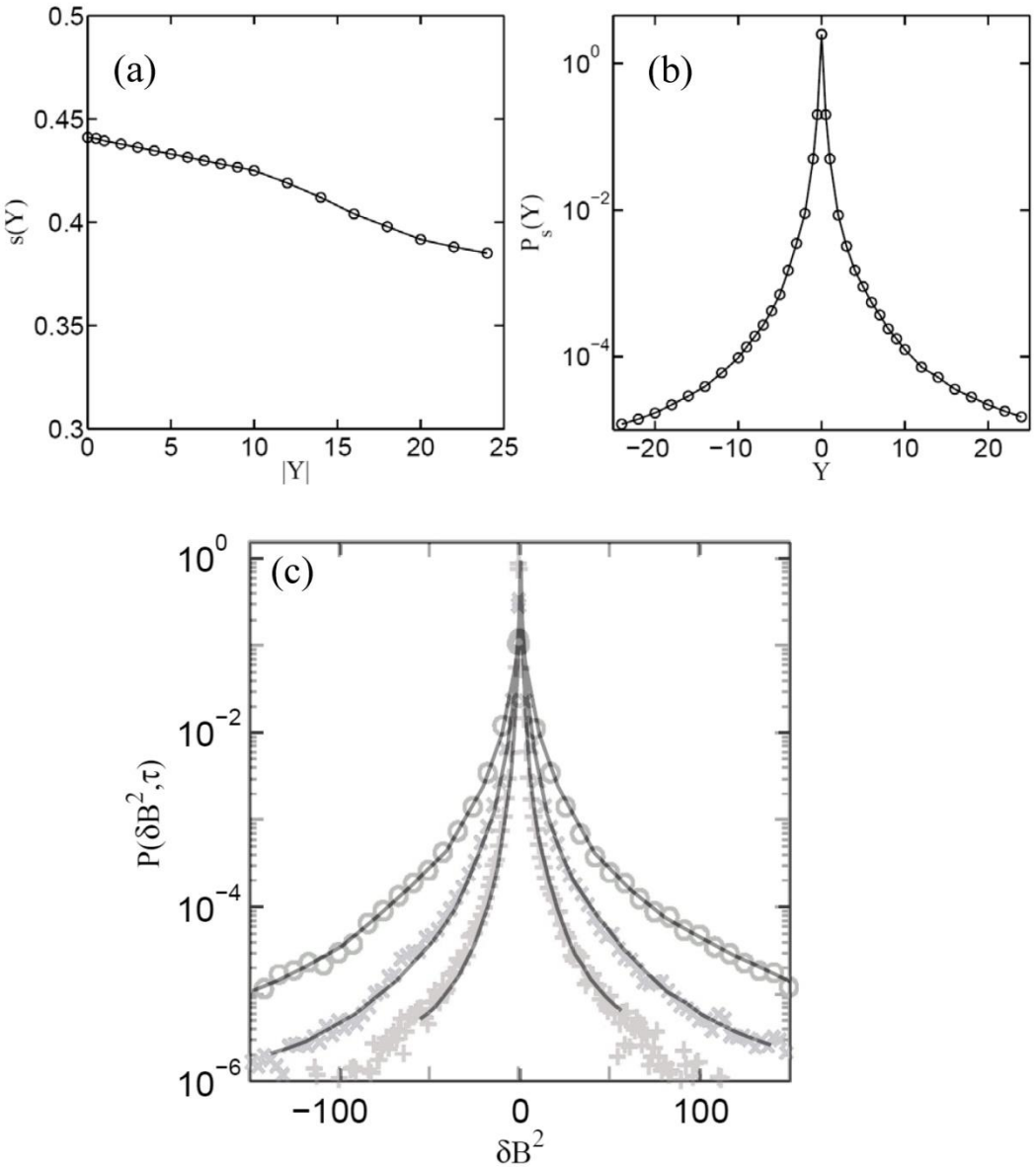




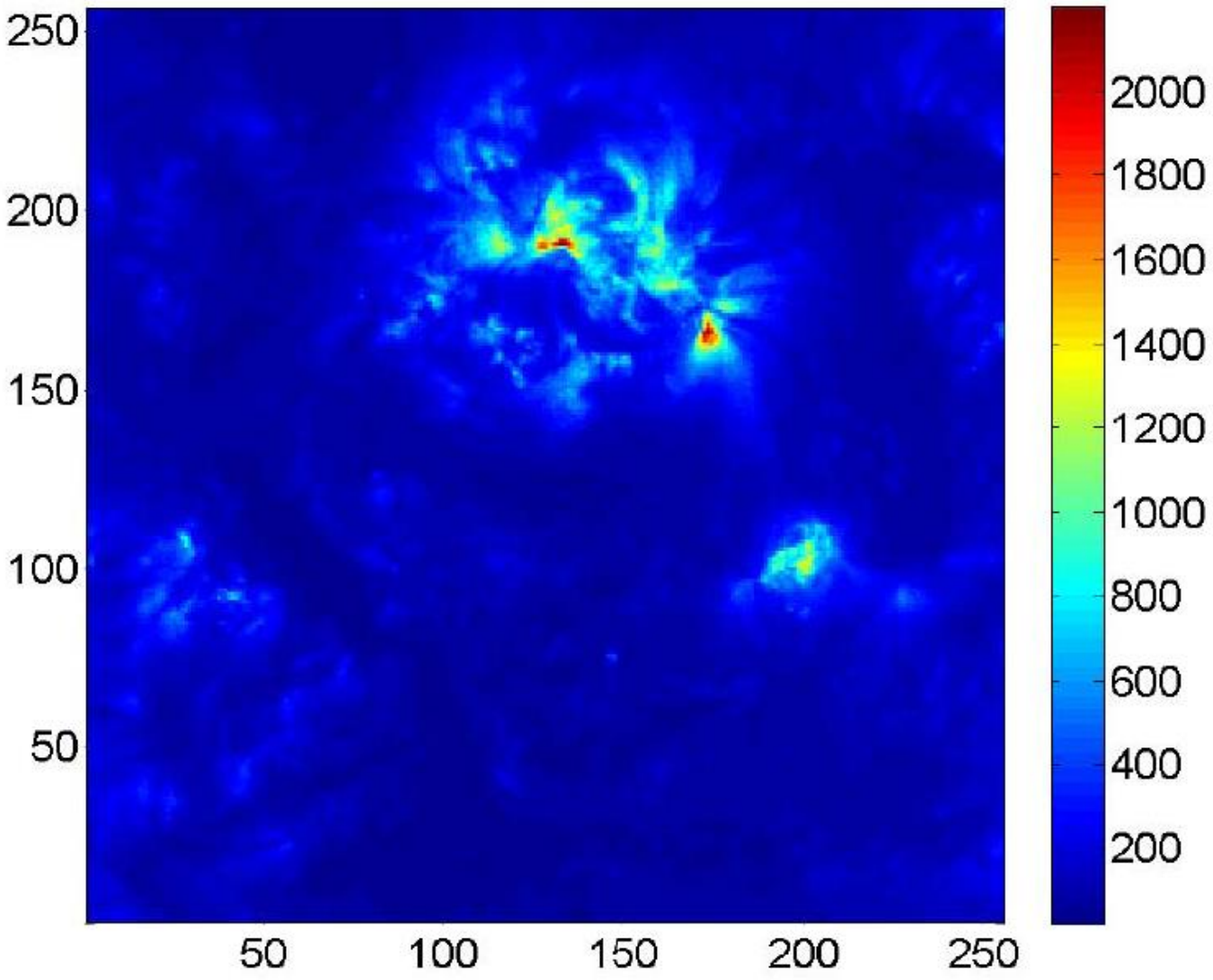
ROMA

(Rank Ordered Multifractal Analyses)

Chang and Wu, 2008

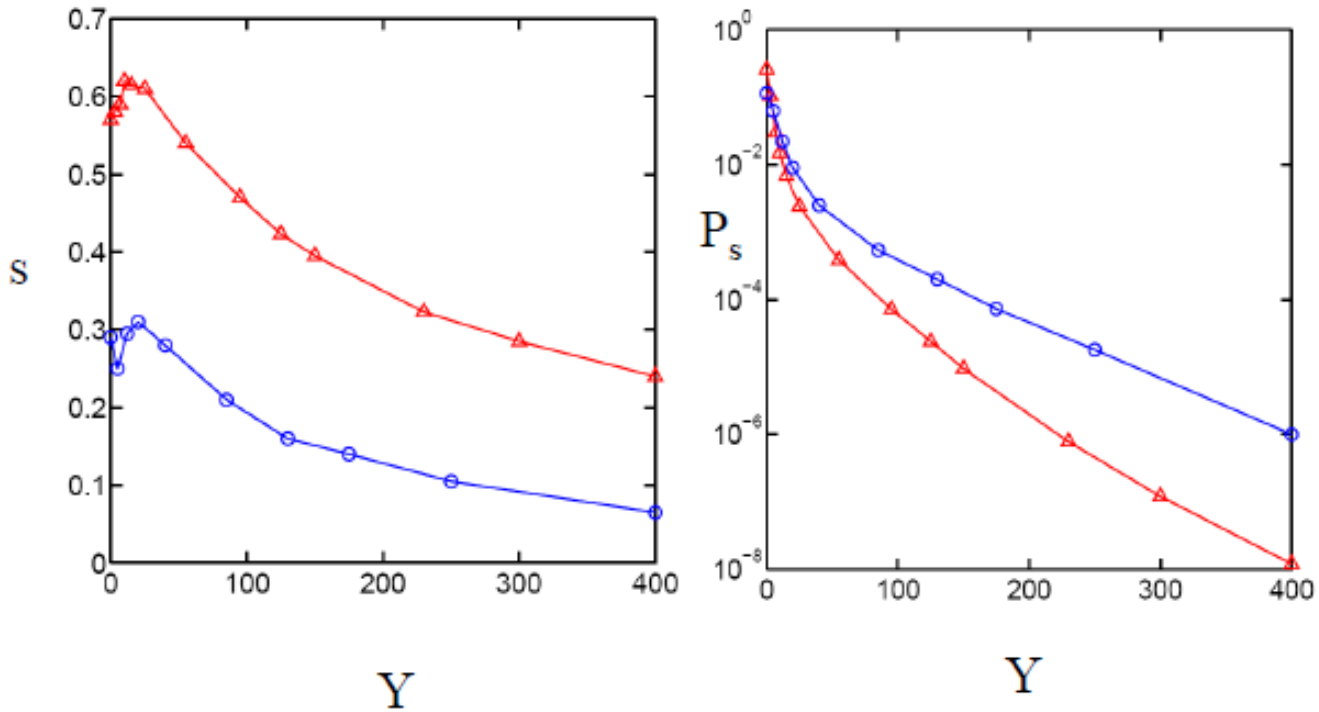

(a)
s(Y)
|Y|
0.5
0.45
0.4
0.35
0.3
0
5
10
15
20
25
(b)
$P_s(Y)$
Y
10^0
10^{-2}
10^{-4}
−20
−10
0
10
20
(c)
$P(\delta B^2,\tau)$
δB^2
10^0
10^{-2}
10^{-4}
10^{-6}
−100
0
100

Corona EUV Emissions – SOHO (Wu et al., 2011)



Wu, Chang, Uritsky; 2011

ROMA analysis of probability distributions for the SOHO EIT data II



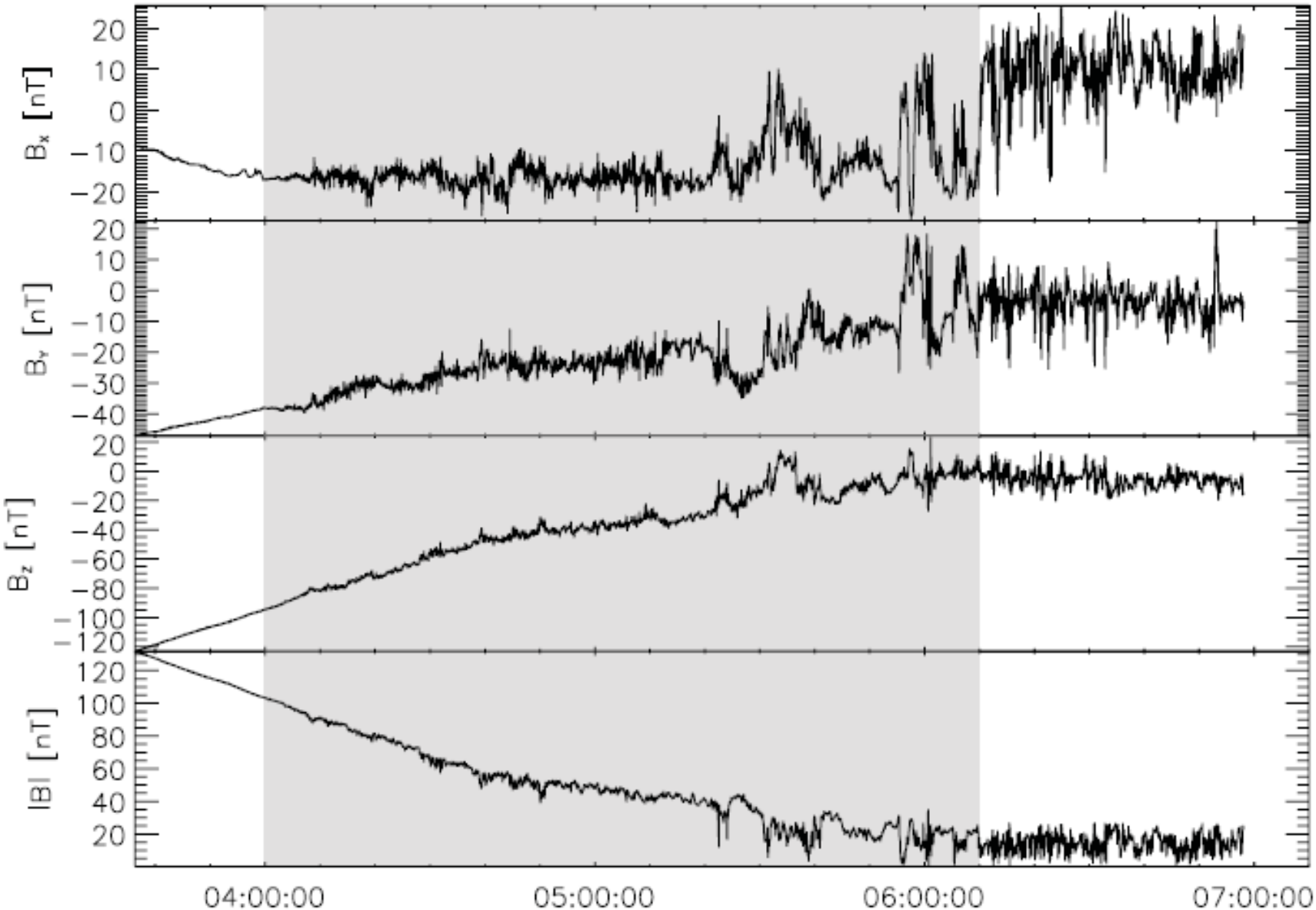
Red triangles: δ from 2 to 8; blue circles: δ from 20 to 80.

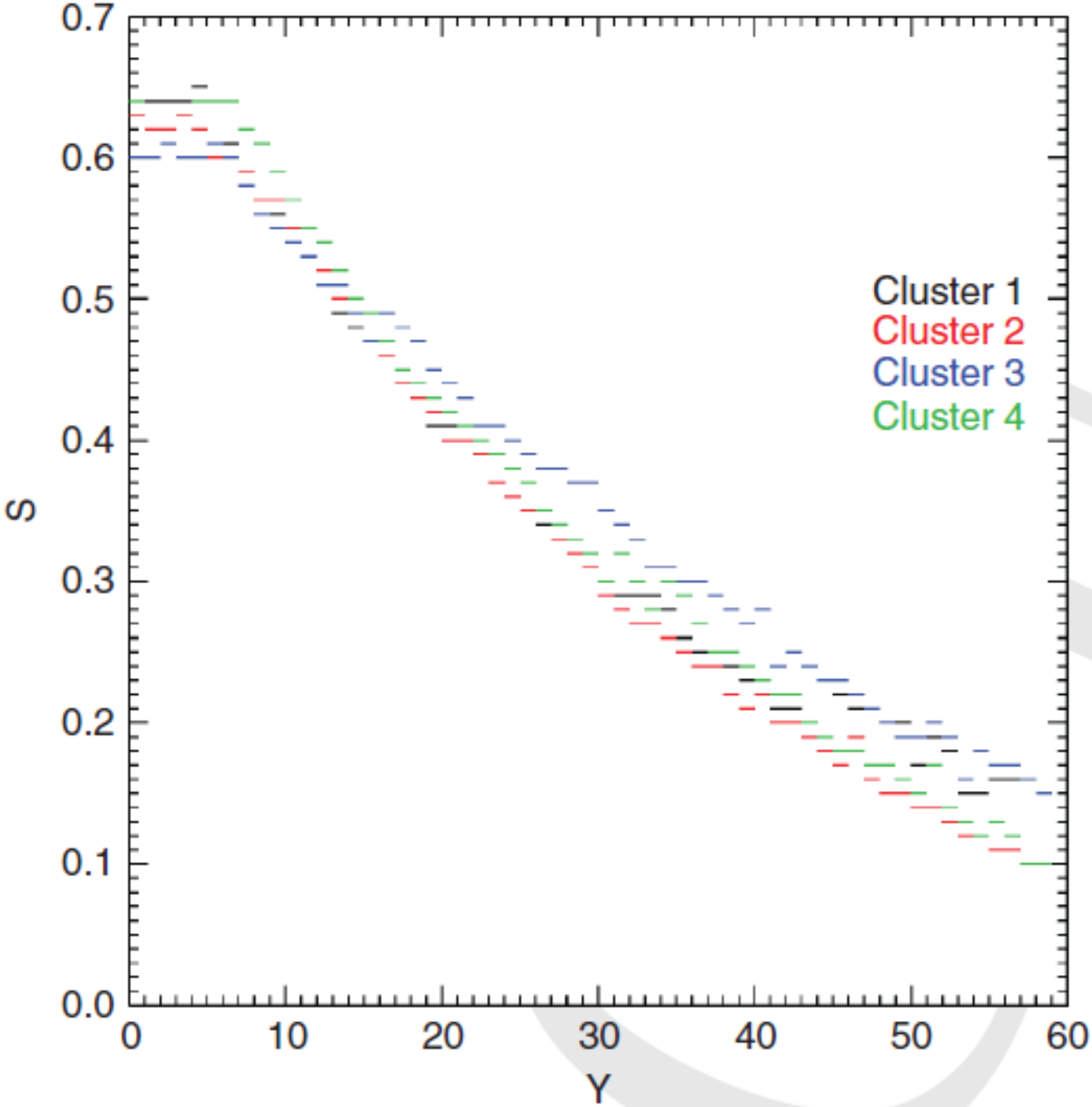
In this study, the SOHO EIT data set is shown to include two multifractal rank-ordered regimes, one for δ from 2 to 8 and the other for δ from 20 to 80.

APPLICATIONS TO SPACE PLASMAS

- Auroral electric field fluctuations (Tam et al.)
- Cusp and magnetosheath magnetic field fluctuations (Echim and Lamy)
- AE index (Consolini and De Michelis)
- EUV solar emissions (Wu and Uritsky)
- Solar wind (Podesta)
- Isotropic hydrodynamic turbulence (Wu)
- Cosmic Web (Chang et al.)

CUSP-Cluster data (Echim et al., 2007 and Lamy et al., 2008)

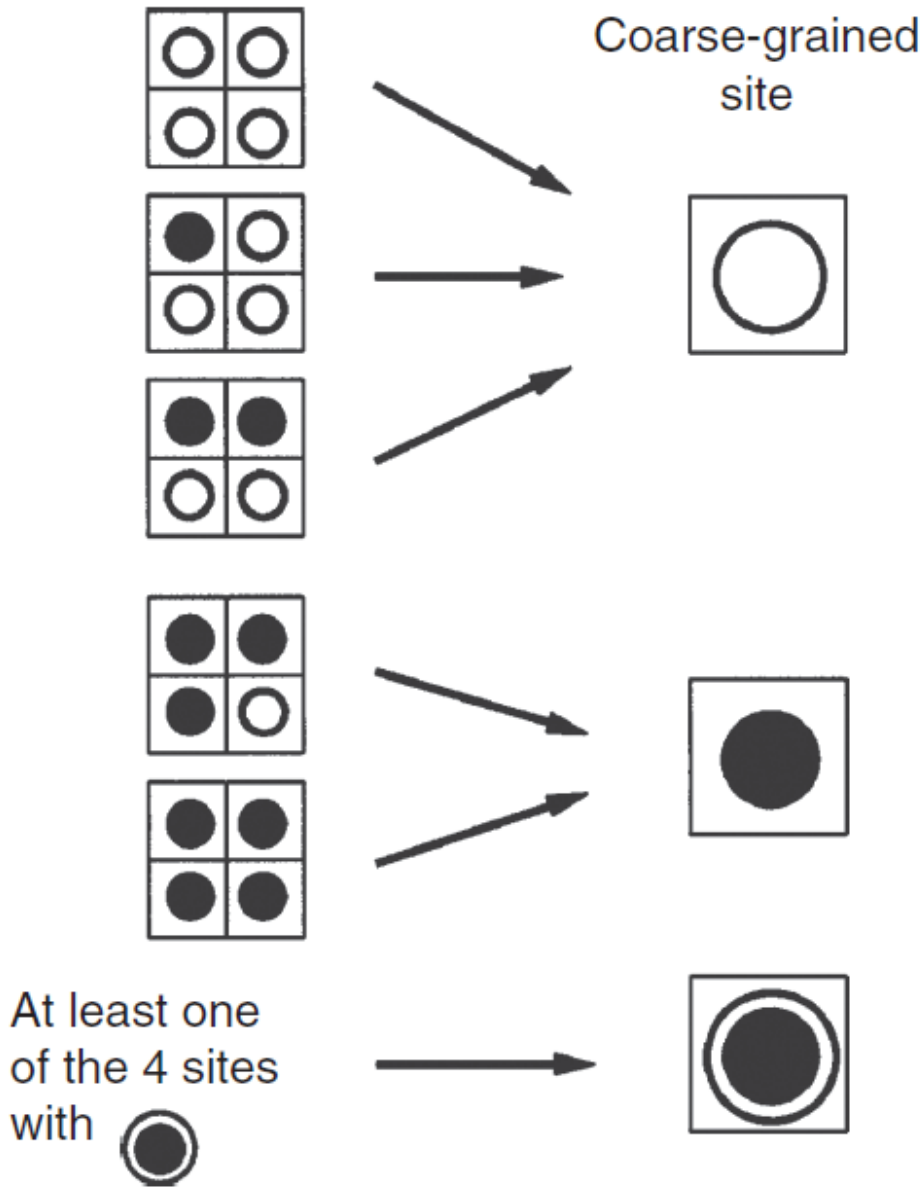



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
S
0
10
20
30
40
50
60
Y
Cluster 1
Cluster 2
Cluster 3
Cluster 4

MODIFIED FOREST FIRE MODEL

Drossel and Schwabl, 1994
Loreto et al., 1995
Tam, Chang, Consolini, De Michelis, 2000

Rules of RG Transformation



Phase Space Global RG Trajectories